# The Influence of the Viewing Geometry of Bare Rough Soil Surfaces on Their Spectral Response in the Visible and Near-Infrared Range

Jerzy Cierniewski

Adam Mickiewicz University, Institute of Physical Geography, Poland

This work is a supplement to the article "A model for soil surface roughness influence on the spectral response of bare soils in the visible and near-infrared range" [Remote Sens. Environ. 23:97–115 (1987)]. It contains new equations necessary to calculate the shadowing coefficient of bare rough soil surfaces observed by a sensor from arbitrary angles and directions.

#### INTRODUCTION

The influence of the degree of soil surface roughness and the solar illumination of soil (defined by the solar altitude, the angle of a slope, and the sloping of the soil surface relative to the sunbeam direction) on soil reflectance in the visible and near-infrared range has been discussed in this journal (Cierniewski, 1987). This influence has been modeled with the assumption that soil reflectance is strongly correlated with the self-shadowing of soil surface. It only referred to a situation when bare rough soil surfaces are observed by a sensor from the nadir.

The present article is prepared as a supplement to the previous one. It contains new equa-

Address correspondence to Jerzy Cierniewski, Adam Mickiewicz University, Institute of Physical Geography, ul. Fredry 10, 61-701 Poznan, Poland.

Received 26 July 1988; revised 18 October 1988.

tions which enable the calculation of the shadowing of soil surfaces observed by a sensor from any direction above simulated soils. The position of the sensor is defined by the zenithal observation angle and the observation direction in relation to the solar direction.

#### NEW EQUATIONS

The unit of area  $(A_u)$  in Eq.  $(1^*)$ , describing the roughness factor  $(RF_m)$  of the simulated structure (Fig. 1), is defined as

$$A_u = d^2 \cos \gamma_s, \qquad (1^*)$$

where  $\gamma_s$  = algebraic sum of the zenith observation angle ( $\theta$ ) and slope angle ( $\gamma$ ). The  $\gamma_s$  value depends on the relationship of the observation direction of the simulated soil surface relative to the solar direction ( $D_{o/s}$ ) and the relation of the  $\theta$  and  $\gamma$  angles ( $R_{\theta/\gamma}$ ) (see Table 1). Thus, the distance between spheres (d) [Eq. (2\*)] is given by the equation:

$$d = \frac{\phi}{2} \left( \frac{\pi}{RF_m} \cos \gamma_s \right)^{1/2}. \tag{2*}$$

<sup>1</sup>Numbers of equations with asterisks follow those from the previous paper (Cierniewski, 1987).

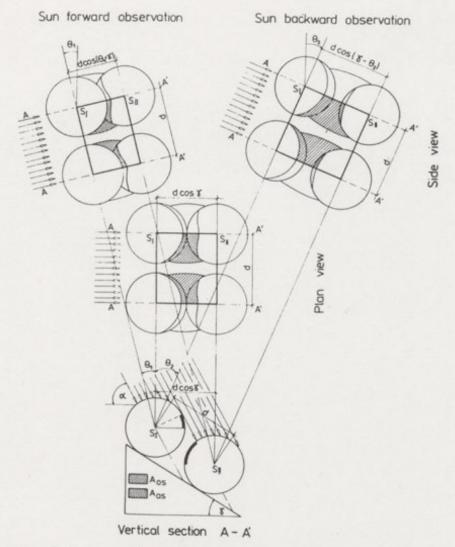


Figure 1. Geometry of the simulated soil surface shadow for calculation of the roughness factor  $(RF_m)$  and the shadowing coefficient  $(SC_m)$ .

The distance of the shadow ellipse center from the given sphere center  $(d_e)$  and the major axis of the shadow ellipse  $(a_e)$  in a view corresponding to any observation direction depends not only on the slope of the soil surface relating to the solar direction (S) as in Eqs. (4\*) and (5\*), but also on  $D_{o/s}$ 

Table 1.

		$R_{\theta/\gamma}$
$D_{o/s}$	$\theta \geqslant \gamma$	$\theta < \gamma$
C	$\gamma_s = \theta + \gamma$	$\gamma_s = \theta + \gamma$
0	$\gamma_s = \theta - \gamma$	$\gamma_s = \theta - \gamma$

C = forward  $D_{o/s}$  (observation direction convergent with sunbeam direction) and  $O = \text{backward } D_{o/s}$  (observation direction opposite to solar direction). and  $R_{\theta/\gamma}$  (Fig. 2). They are expressed by the formulas in Table 2.

The  $A_c$  shadow component from Eq. (8\*) depends on  $D_{o/s}$  and the relation of the  $\theta$  and  $\alpha$  angles  $(R_{\theta/\alpha})$  (see Table 3). The  $r_{\min}$  formula (Fig. 3) was found from the equation

$$y = \pm \tan \alpha_s x + \frac{r}{\cos \alpha_s} + d(\pm \sin \gamma_s \pm \cos \gamma_s \tan \alpha_s),$$

where  $\alpha_s$  = algebraic sum of zenith observation angle  $(\theta)$  and solar altitude angle  $(\alpha)$ .  $\alpha_s$  depends on  $D_{\sigma/s}$  and  $R_{\theta/\alpha}$  (see Table 4). The  $r_{\min}$  value also depends on the sloping (S) and the  $R_{\theta/\gamma}$  relation (see Table 5). The choice of the proper formula describing the position of the  $X_{ri}$  point from the equations defined as in Table 6 should

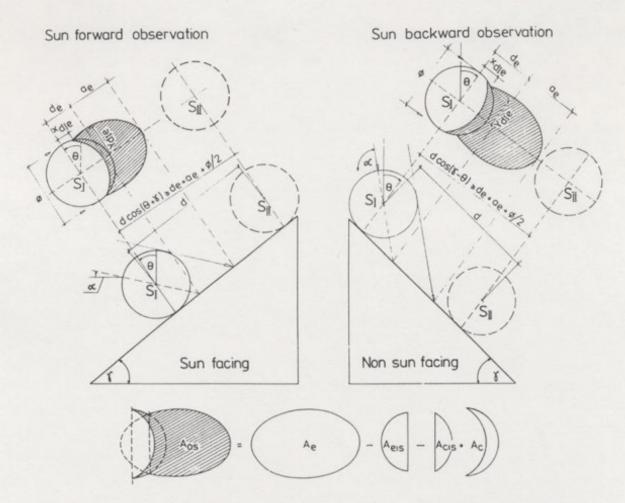


Figure 2. Geometry of an own-sphere shadow and a shadow on a slope plane cast by a given sphere.

Table 2

able 2.				
S	$D_{o/s}$	$R_{\theta/\gamma}$	Element of Shadow Area	
F	С	$\theta \geqslant \gamma$	$d_e = \frac{\phi}{2} \cos \gamma_s \left[ \tan(90 - \alpha - \gamma) - \tan \gamma_s \right]$	(4a*)
F	C	$\theta < \gamma$		
F	0	N )	$d_e = \frac{\phi}{2} \cos \gamma_s \left[ \tan (90 - \alpha - \gamma) + \tan \gamma_s \right]$	(4b*)
F	N	N	$a_e = \frac{\phi}{2} \frac{\cos \gamma_s}{\cos (90 - \alpha - \gamma)}$	(5a*)
В	C	N )		
В	0	$\theta < \gamma$	$d_e = \frac{\Phi}{2} \cos \gamma_s \left[ \frac{1}{\tan(\alpha - \gamma)} - \tan \gamma_s \right]$	(4c*)
В	0	$\theta \geqslant \gamma$	$d_e = \frac{\phi}{2} \cos \gamma_s \left[ \frac{1}{\tan(\alpha - \gamma)} + \tan \gamma_s \right]$	(4d*)
В	С	$\theta \geqslant \gamma$	$a_e = \frac{\phi}{2} \cos \gamma_s \left[ \frac{1 + \cos(\alpha - \gamma)}{\sin(\alpha - \gamma)} - \tan \gamma_s \right]$	(5b*)
В	C	$\theta < \gamma$		
В	0	N }	$a_e = \frac{\phi}{2} \frac{\cos \gamma_s}{\sin(\alpha - \gamma)}$	(5c*)

F = forward (sun-facing) slope, B = backward (non-sun-facing) slope, N = nonlimiting  $R_{\theta/\gamma}$ relation,  $\alpha$  = solar altitude.

Table 3.

$D_{o/s}$	$R_{\theta/\alpha}$	Element of Shadow Area	
С	N	$A_r = \pi \frac{\phi^2}{8} \left[ 1 - \sin(\theta + \alpha) \right]$	(8a*)
0	$\theta \geqslant \alpha$	$A_{c} = \pi \frac{\phi^{2}}{8} \left[ 1 + \sin(\theta - \alpha) \right]$	(8b*)
0	$\theta < \alpha$	$A_c = \pi \frac{\phi^2}{8} \left[ 1 - \sin(\alpha - \theta) \right]$	(8c*)

take into account the influence of the same factors as above.

The shadow area fragment  $A_{sc}$  is given by the new formula:

$$A_{sc} = \frac{1}{2} \left[ \phi^2 \arccos \frac{d \cos \gamma_s}{\phi} - d \cos \gamma_s (\phi^2 - d^2 \cos \gamma_s)^{1/2} \right], \quad (19^*)$$

with the limitation that  $A_{sc}$  only refers to the backward (non-sun-facing slope when  $\phi > d \cos \gamma_s$ .

## RESULTS

The results of the soil surface shadowing simulation by the new equations are presented on exam-

Table 4.

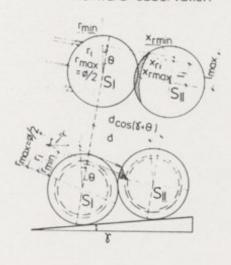
		$R_{\theta/\alpha}$
$D_{o/s}$	$\theta \geqslant \alpha$	$\theta < \alpha$
C	$\alpha_s = \theta + \alpha$	$\alpha_s = \theta + \alpha$
0	$\alpha_s = \theta - \alpha$	$\alpha_s = \alpha - \theta$

ples of observation conditions referring to the zenith observation angles  $(\theta)$  of  $0^{\circ}$  (nadir),  $10^{\circ}$ , and  $30^{\circ}$  both for forward and backward observation directions (Fig. 4). The shadowing coefficient of soil surface (SC<sub>m</sub>) reaches values from 1 to 0. The SC<sub>m</sub> value of 1 means a complete shadowing of soil surfaces on backward slopes illuminated by direct sunbeams falling at a lower solar altitude  $(\alpha)$  than the slope angle  $(\gamma)$ , as follows from the equation

$$\alpha_{SC_{-}=1} \leq \gamma$$
. (1)

The value of 0 refers to situations when the shadowed fragments of an illuminated soil are not visible to a sensor from a given direction of its observation. The shadowing coefficient is 0 if the soil surfaces are observed from the direction convergent to the solar direction. The greater the zenith observation angle  $(\theta)$ , the lower the solar

Sun forward observation



Sun facing

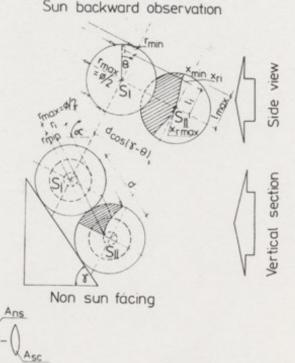


Figure 3. Geometry of a shadow on a sphere cast by an adjoining sphere.

Table 5.

			Relation of Angles			
	Element of Shadow Area	$\theta/\alpha$	θ/γ	$D_{\sigma/s}$	S Doja	
(14a*)	$r_{\min} = d \cos \alpha_s \frac{-\sin \gamma_s + \cos \gamma_s \tan \alpha_s}{1 + \sin \alpha_s}$	N	$\theta \geqslant \gamma$	С	F	
(14b*)	$r_{\min} = d \cos \alpha_s \frac{\sin \gamma_s + \cos \gamma_s \tan \alpha_s}{1 + \sin \alpha_s}$	N	$\theta < \gamma$	С	F	
(14c*)	$r_{\min} = d \cos \alpha_s \frac{\sin \gamma_s - \cos \gamma_s \tan \alpha_s}{1 - \sin \alpha_s}$	$\theta \geqslant \alpha$	N	0	F	
	as (14b*)	$\theta < \alpha$	N	0	F	
	as (14a*)	N	N	C	B B	
	as (14c*)	$\theta \geqslant \alpha$	$\theta \geqslant \gamma$	0	В	
(14d*)	$r_{\min} = d \cos \alpha_x \frac{-\sin \gamma_s - \cos \gamma_s \tan \alpha_s}{1 - \sin \alpha_s}$	$\theta \geqslant \alpha$	$\theta < \gamma$	0	В	
	as (14b*)	$\theta < \alpha$	$\theta \geqslant \gamma$	0	В	
	as (14a*)	$\theta < \alpha$	$\theta < \gamma$	0	В	

Table 6.

		Relation of Angles				
S	$D_{o/s}$	θ/γ	$\theta/\alpha$	Element of Shadow Area		
F	C	$\theta > \gamma$	N	$X_{ri} = -\cos^2 \alpha_s \left[ \frac{r_i \tan \alpha_s}{\cos \alpha_s} - dM \tan \alpha - \left( \frac{2r_i dM}{\cos \alpha_s} - d^2M^2 \right)^{1/2} \right]$		
F	C	$\theta < \gamma$	N	$M = -\sin \gamma_s + \tan \alpha_s \cos \gamma_s$ $X_{ri} = -\cos^2 \alpha_s \left[ \frac{r_i \tan \alpha_s}{\cos \alpha_s} - dM \tan \alpha - \left( \frac{2r_i dM}{\cos \alpha_s} d^2 M^2 \right)^{1/2} \right]$ (16a*)		
F	0	N	$\theta \geqslant \alpha$	$M = \sin \gamma_s + \tan \alpha_s \cos \gamma_s$ (16b* $X_{ri} = -\cos^2 \alpha_s \left[ \frac{-r_i \tan \alpha_s}{\cos \alpha_s} + dM \tan \alpha - \left( \frac{2r_i dM}{\cos \alpha_s} - d^2 M^2 \right)^{1/2} \right]$		
F	0	N	$\theta < \alpha$	$M = \sin \gamma_x - \tan \alpha_x \cos \gamma_x $ (16e*)		
В	С	N	N	$X_{ri} = -\cos^2 \alpha_s \left[ \frac{r_i \tan \alpha_s}{\cos \alpha_s} + dM \tan \alpha - \left( \frac{-2r_i dM}{\cos \alpha_s} - d^2 M^2 \right)^{1/2} \right]$		
В	0	$\theta \geqslant \gamma$	$\theta \geqslant \alpha$	$M = -\sin \gamma_s - \tan \alpha_s \cos \gamma_s $ $X_{ri} = -\cos^2 \alpha_s \left[ \frac{-r_i \tan \alpha_s}{\cos \alpha_s} - dM \tan \alpha - \left( \frac{-2r_i dM}{\cos \alpha_s} - d^2M^2 \right)^{1/2} \right]$ $M = -\sin \gamma_s - \tan \alpha_s \cos \gamma_s $ $M = -\cos^2 \alpha_s \left[ \frac{-r_i \tan \alpha_s}{\cos \alpha_s} - dM \tan \alpha - \left( \frac{-2r_i dM}{\cos \alpha_s} - d^2M^2 \right)^{1/2} \right]$		
В	0	$\theta < \gamma$	$\theta \geqslant \alpha$	$M = -\sin \gamma_s + \tan \alpha_s \cos \gamma_s$ $\chi_{ri} = -\cos^2 \alpha_s \left[ \frac{-r_i \tan \alpha_s}{\cos \alpha_s} - dM \tan \alpha - \left( \frac{-2r_i dM}{\cos \alpha} - d^2 M^2 \right)^{1/2} \right]$		
В	0	$\theta < \gamma$	$\theta < \gamma$	$M = \sin \gamma_x + \tan \alpha_x \cos \gamma_x$ as (16d*)		
В	0	$\theta \geqslant \gamma$	$\theta < \alpha$	$X_{ri} = -\cos^2 \alpha_s \left[ \frac{-r_i \tan \alpha_s}{\cos \alpha_s} + dM \tan \alpha - \left( \frac{-2r_i dM}{\cos \alpha_s} - d^2 M^2 \right)^{1/2} \right]$		
				$M = -\sin\gamma_s - \tan\alpha_s \cos\gamma_s \qquad (16g^*)$		

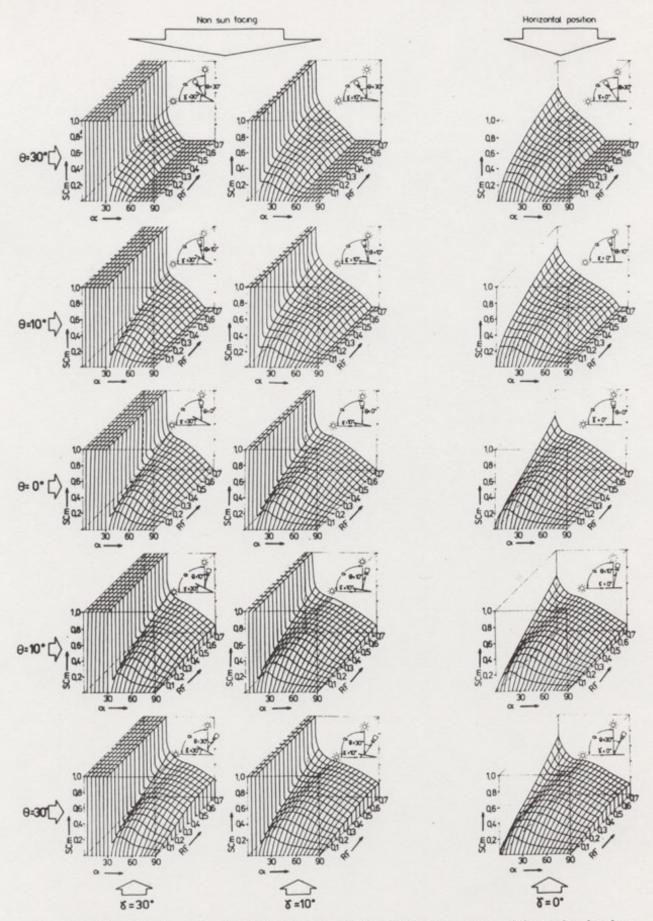


Figure 4. Relationship between the simulated shadowing coefficient of soil surface ( $SC_m$ ) and the rough soil surface factor ( $RF_m$ ), the solar altitude ( $\alpha$ ), the slope angle of soil surface ( $\gamma$ ), and the zenithal observation angle ( $\theta$ ).

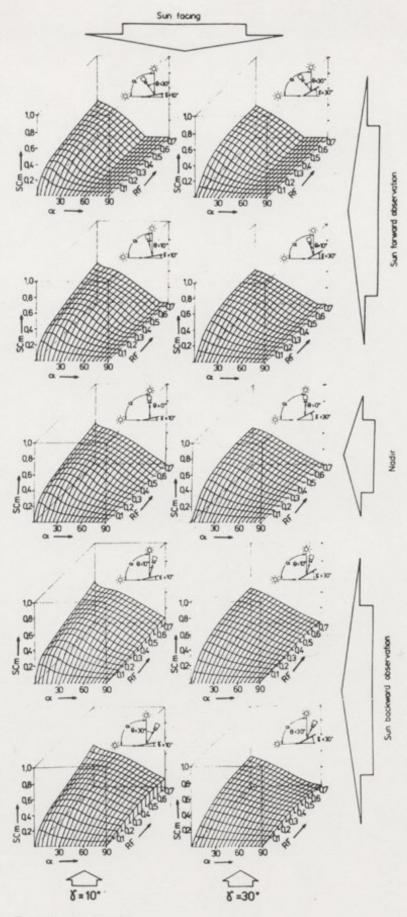


Figure 4. (continued)

Table 7. Soil Shadowing Coefficient ( $SC_m$ ) for Different Illumination and Observation Conditions on the Example of Soil Surface Example with the Roughness Factor  $RF_m = 0.3$ 

Solar Altitude	Forward C	bservation	Nadir	Backward Observatio	
$(\alpha^0)$	$\theta = 30^{\circ}$	$\theta = 10^{\circ}$	$\theta = 0^{\circ}$	$\theta = 10^{\circ}$	$\theta = 30^{\circ}$
	For	ward (Sun-Facir	ig) Slope $\gamma = 3$	0°	
15	0.28	0.33	0.34	0.33	0.29
30	0.15	0.23	0.26	0.27	0.24
45	0.06	0.13	0.16	0.19	0.2
60	0	0.06	0.09	0.12	0.15
75	0	0.01	0.04	0.07	0.11
90	0	0	0.02	0.02	0.06
		Horizontal Pos	ition $\gamma = 0^{\circ}$		
15	0.35	0.4	0.37	0.37	0.36
30	0.28	0.35	0.36	0.38	0.38
45	0.16	0.29	0.28	0.31	0.34
60	0	0.16	0.15	0.2	0.26
75	0	0.03	0.06	0.1	0.17
90	0	0	0	0.03	0.09
	Backw	ard (Non-Sun-Fe	icing) Slope y	= 30°	
15	1	1	1	1	1
30	1	1	1	1	1
45	0.19	0.29	0.32	0.34	0.37
60	0	0.19	0.25	0.3	0.36
75	0	0.04	0.11	0.18	0.28
90	0	0	0.05	0.05	0.15

altitude ( $\alpha$ ) under which the shadowing parameter ( $SC_m$ ) reaches 0:

$$\alpha_{SC_m=0} \geqslant 90 - \theta. \tag{2}$$

The formula states that for observation from the nadir  $SC_m$  does not reach 0 before the sun zenith position.

Rough soil surfaces observed from a backward direction to sunbeams are more shadowed than those observed from the nadir and a lot more shadowed than those seen from a forward direction. When looking at the rough soil from the backward direction, its shadowing coefficient  $(SC_m)$  increases with an increase of the zenith observation angle  $(\theta)$ , whereas when looking at the same soil, but from the forward direction, this relation is opposite.

The SC<sub>m</sub> coefficient differentiation resulting from the influence of the soil observation direction, analyzed on the example of a soil surface of average roughness for cultivated soils (Table 7), is clear. This influence is greater at a higher solar altitude, especially for the sun level  $(\alpha)$  which satisfies the equation

$$\alpha \geqslant 90 - \theta$$
. (3)

The shadowing coefficient of soil surface ( $SC_m$ ) for the zenith observation angle of 30° can even differ by more than 100% from the  $SC_m$  value in relation to the observation from the nadir direction.

The equations presented in this paper, describing the influence of observation conditions on the shadowing of soil surfaces observed by a sensor, not only can find any function in the remote sensing of soils in the visible and near-infrared range of the spectrum, but also may be useful in the microwave range.

## REFERENCE

Cierniewski, J. (1987), A model for soil surface roughness influence on the spectral response of bare soils in the visible and near-infrared range, Remote Sens. Environ. 23:97-115.