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Vector algebra for Steep Slope Model analysis

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Abstract: Geographic Information Systems offer many algorithms that allow analysis of digital elevation models. They work with both GRID and TIN data, but they are limited to 2.5D models, where one planar (X,Y) position refers to only one vertical (Z) value. In mountainous regions, however, many steep, vertical and even overhung parts of rock walls and slopes occur. GRID and TIN models in a standard projection are not capable to deal with such a relief as they are not able to capture all complexity of steep slopes that can be observed from the terrestrial perspective. Such a perspective can be introduced into GIS via computer graphics software that allows 3D surface modelling by means of *mesh*, e.g. 3D triangular network. The paper presents a concept that implements 3D mesh in GIS and utilizes vector algebra to analyze such a surface. The idea is based on using normal vectors to compute slope and aspect of each triangle in a mesh. The computed values are saved as their attributes. Complete procedures are written in Python programming language and implemented into popular GIS software to work as a plug-in tool.

Keywords: vector algebra, Steep Slope Model, Geographic Information Systems, slope, aspect

Introduction

One of the classical problems in geomorphology are the topics of slope evolution and its quantitative description (Klimaszewski 1978, Gerrard 1990, Bishop & Schroder 2004). In mountainous regions many steep and even overhung parts of rock walls and slopes can be observed. They suffer from such hazardous events as rockfalls, landslides, debris flows or avalanches and the effects of the phenomena can be visible in the landscape for a long time. Due to landscape dynamics, however, the relief can change rapidly. For that reason, assessment of changes of the affected forms demands better recognition and constant monitoring (Bishop & Schroder 2004, Rączkowska 2006). Klimaszewski (1978) points out that slope systems control structure and functioning of the landscape. Slope gradient and aspect allow terrain characterization and are important parameters in many surface processes, such as mass movements or water runoff (Willson & Gallant 2000).

Many geomorphologic studies exploit terrain parameters derived from Digital Elevation Model (DEM) analysis. DEM is a numerical representation of the terrain surface composed of a set of points located on the terrain surface (Gaździcki 1990, Li et al. 2005). The definition includes also algorithms that allow reconstructing the shape of the surface (Gaździcki 1990). DEM is one of the fundamental components of geodatabases, playing an important role in Geographic Information Systems (GIS) and environmental modelling. It is widely applied in many disciplines such as: mapping, remote sensing, engineering, geology, geomorphology, spatial planning. The currently available DEMs are mostly derived from photogrammetric, laser scanning or radar measurements. Some of them, however, are products of ground surveys or contour digitizing on topographic maps (Li et al. 2005).

Digital elevation data can be organized into two various data structures, depending mainly on the preferred method of storage and analysis: regular grids (rasters) and triangulated irregular networks (TIN). The regular grids, due to their simplicity, have become the most widely used data structure (Willson & Gallant 2000; Li et al. 2005). The matrix notation of elevation values enables a great number of mathematical operations based on topological relations between neighbouring data points. The mathematical computations can be easily coded and implemented into GIS software. Regular grids, however, have also disadvantages. Constant square grids are not flexible enough to describe abrupt elevation changes and relief details. High resolution grids consume a lot of memory for analysis and storage (Lane et al. 2000). For that reason, a specific trade-off must be found between levels of detail and a dataset size. The problems have been partially solved by quadtree models (Agrawal et al. 2006), which allow DEM densification in particular areas and storage space reduction (Rahman 1994). An interesting concept of lattice was introduced in ARC/INFO workstation. Both lattices and grids are stored in the same data structure, but the regularly distributed lattice points are interpreted as surface values at the centre of a cell, which does not force an area to have a constant value (ESRI 2012).

Triangulated irregular networks are popular due to the fact that they incorporate surface-specific points and lines (e.g. peaks, pits, breaklines, fault lines), and the density can vary to match the surface roughness (Weibel & Heller 1991, Li et al. 2005). The classical TIN creation algorithm is Delaunay triangulation, satisfying the criterion that any vertex can lay inside the circumcircle of the triangles in the network, and thin triangles are avoided by maximizing minimum interior angles. Significant disadvantage of TIN, as compared to grid DEMs, is a lower processing efficiency and a more complicated computation of terrain parameters.

In the traditional DEM the terrain information is represented by 2D locations with unique mathematical attribute of altitude, e.g. one planar (X,Y) position refers to only one vertical (Z) value. Such simplified depiction of terrain is called 2.5D (Weibel 1993, Schön et al. 2009). With increasing slope gradient, the same distance between two adjacent grid cells or TIN vertices represents increasing real distances along the steep slope surfaces, hence leading to significant simplification and loss of detail. Such a perspective can be introduced via computer graphics software that allows 3D surface modelling by means of a mesh (e.g. 3D triangular network) or lattice, which is introduced as Steep Slope Model (SSM) (Buchroithner 2002, Kolecka 2012).

In 2.5D conceptual model, both grid and TIN surface models can be analysed with ready-to-use tools implemented into GIS software. As far as slope gradient is concerned, it is defined as the inclination angle between the surface and a horizontal plane. To obtain results in degrees, slope is calculated as arctangent of the change in height divided by the change in horizontal distance (ESRI 2012). In most applications the grid model is used, and the results of calculation depend on its resolution. Usually, a 3×3 kernel (window) is moving along the grid in x and y direction to compute value for each cell. The method, however, tends to underestimate slope when using low-resolution grids or on rough surfaces (Corripio 2003). Contrary to 2.5D, the 3D surface models processing in GIS, especially GIS analysis of mountainous surface models and geomorphic relief description, is impossible or limited (Schön et al. 2009).

The above considerations were an inspiration to formulate the main goal of this study: to develop a method that introduces SSM into GIS and to provide tools (algorithms) to perform SSM analysis. The goal was achieved by integration of computer graphic, vector algebra and Python (programming language) scripting. The concept of vector algebra use in computation of terrain parameters was introduced with lattices. The similar concept was already used by Corripio (2003). Both of them, however, were dedicated to 2.5D models. The approach presented in this work considers different type of input data and partially different computational algorithms. The developed methodology was examined on the study area located in the Polish Tatra Mountains, steep and partially overhung slopes of Kościelec Mtn.

In the next section, study area and used datasets are described. Section three presents the state-of-the-art and the methodological approach. In section four results of its implementation are given. The last section contains discussion and conclusions.

Test site and datasets

The mountain range of the Tatras is located at the border between Poland and Slovakia. It represents an alpine landscape and reaches up to 2655 m a.s.l. in the Gerlach peak. The lower parts are built predominantly of carbonate rocks, while in the highest parts granite and metamorphic rocks dominate. The alpine character of the relief results mainly from glacial transformation in the Pleistocene, and later modified by periglacial processes which formed a system of cliffs and talus slopes (Kotarba 1992, Rączkowska 2006). Nowadays, the relief is strongly influenced by slope and fluvial processes (Kotarba et al. 1987, Krzemień 1991, Kotarba & Pech 2002).

The test site was located in the Polish High Tatra Mountains and covered western slopes of the Kościelec Mt. (Fig. 1, 2). The Kościelec massif lies in the northern side-ridge of the Zawratowa Turnia. Its distinctive relief is formed by obliquely sloping beds of granite. The massif consists of two peaks: Wielki Kościelec (2158 m) and Zadni Kościelec (2160 m), separated by the Kościelcowa Pass (2110 m). The western walls of Kościelec are overhung in the lower parts, with talus cones at their feet up to 100 meters high. They are one of the most interesting and most often visited walls by climbers in the Polish Tatra Mountains.

The datasets used in this work were: 3D mesh of the test site (Fig. 3) and a digital elevation model in the traditional TIN format. The original 3D mesh

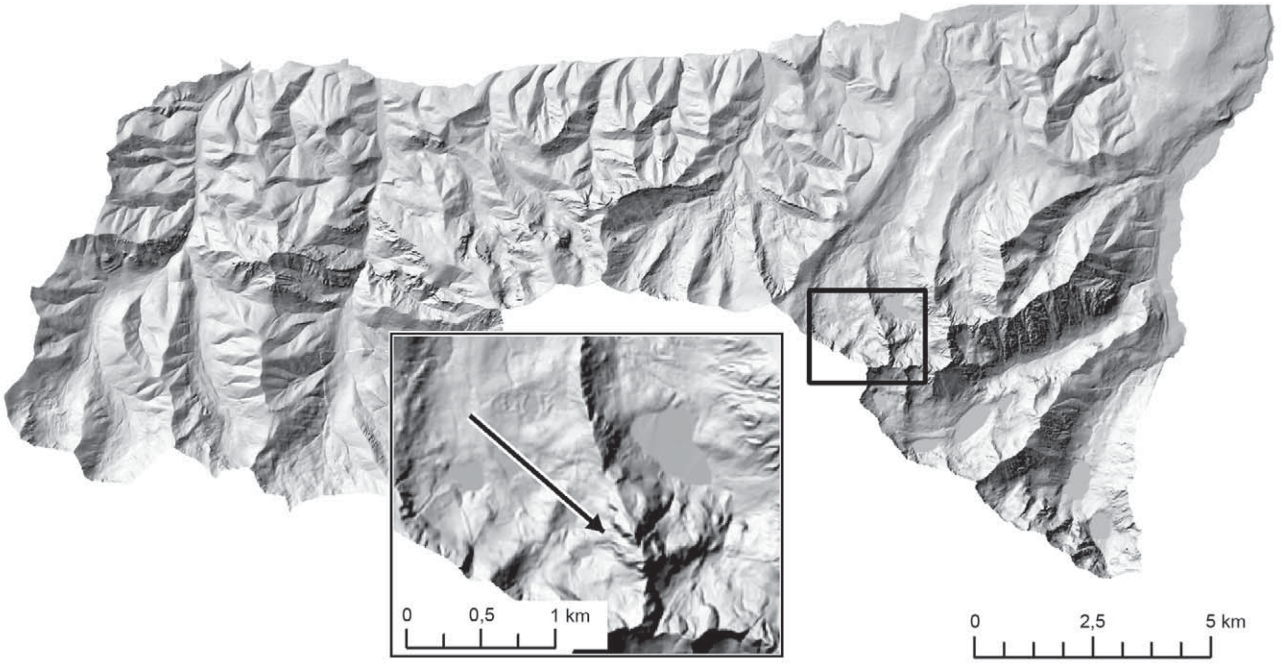

Fig. 1. Shaded relief map of the Polish Tatra National Park; test site and its surroundings are marked and enlarged

Fig. 2. Western slopes of the Kościelec Mt.

was very dense and consisted of over 1 million triangles, produced by means of terrestrial photogrammetry, image matching and dense point cloud generation and modelling (Kolecka 2011). For the analysis, however, a part of it was extracted and decimated, due to limited hardware computational power. The resulting mesh had an average vertex spacing of about 1 m, and the total number of triangles equalled to 20 211.

The TIN, used for comparison, was obtained from the CODGiK (the Main Geodetic and Cartographic Documentation Centre) in Poland. It was produced by stereo-photogrammetric processing of aerial photographs taken in 2009, with terrain resolution of 0.20 m.

Methods

Steep slopes having complex and rough surface are represented efficiently by 3D mesh, composed of irregular triangles. Even overhung parts can be described in that way in computer graphics. Each triangle is considered as the smallest surface unit – a pla-

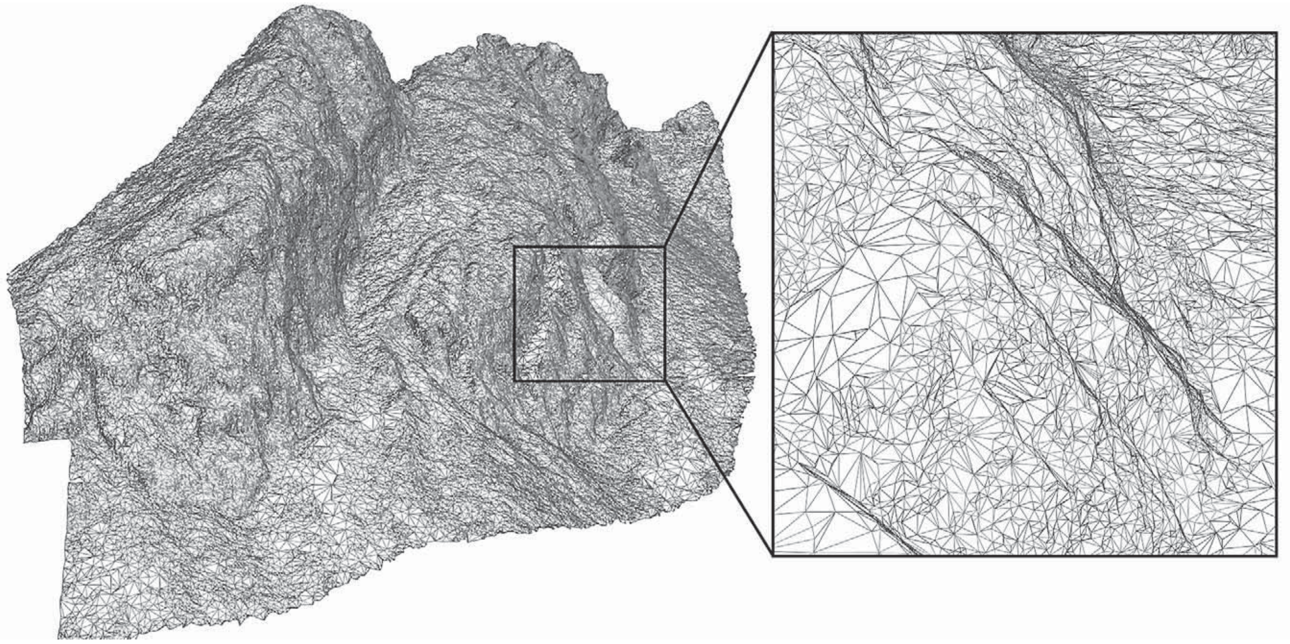

Fig. 3. 3D triangular mesh of the mesh and TIN od the test site (Kolecka, 2011)

ne enclosed by three data points of coordinates x_i, y_i, z_i. The triangular unit ensures exact representation, as any plane is defined by three points in space. That fact is emphasized by Corripio (2003), despite the fact that he considered the smallest reference surface as a cell enclosed by four vertices. To overcome this problem, he divided the cell into two triangles. The different shapes of the smallest surface unit are the secondary aspect. The concept of lattice and grid used in GIS is limited to 2.5D. The crucial thing is the capability to store more than one Z-value for each (X,Y) position, and the 3D mesh has it.

The most frequently computed terrain parameters are slope gradient and orientation. Both can be defined by means of the normal vector **n**, e.g. 3D vector perpendicular to the surface (Hodgson & Gaile 1999) (Fig. 4). According to the mathematical definition, the gradient is defined as a unit normal vector $\mathbf{n_u}$ (Weisstein 2012). A dot product of the surface and its normal vector equals zero. The **n** vector to a plane given by eq. 1 is specified by eq. 2:

$$f(x, y, z) = ax + by + cz + d = 0 \quad (1)$$

$$n = \begin{vmatrix} a \\ b \\ c \end{vmatrix} \quad (2)$$

A plane specified by three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is:

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \quad (3)$$

The normal unit vector can be then immediately written as:

$$n_u = \frac{(x_3 - x_1) \times (x_2 - x_1)}{\left|(x_3 - x_1) \times (x_2 - x_1)\right|} \quad (4)$$

The vector orientation plays an important role as well. As far as the mesh is considered, one should distinguish between inward- and outward- pointing normal vectors. In this work we use the outward-pointing normal vectors. Having the $\mathbf{n_u}$ vector for each reference surface unit, slope (ζ) and aspect (ψ) parameters are calculated according to eq. 5 and 6.

$$\varsigma = \arccos n_{uz} \quad (5)$$

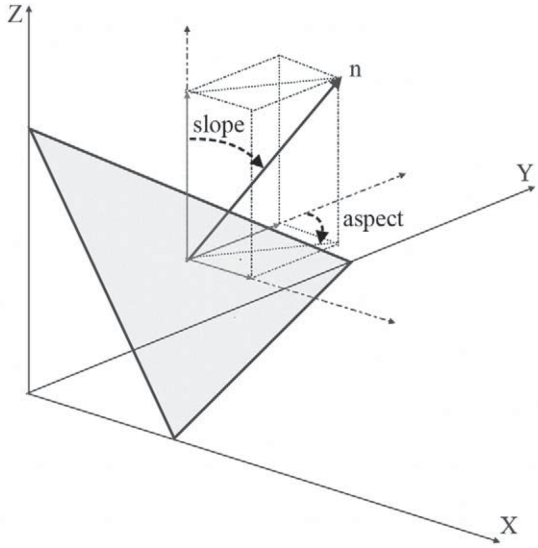

Fig. 4. Normal vector to a plane, slope and aspect angles

$$\psi = \frac{\pi}{2} + \arctan \frac{n_{uy}}{n_{ux}} \qquad (6)$$

where n_{ux}, n_{uy}, n_{uz}, are the $\mathbf{n_u}$ vector components.

The SSMs are built using software dedicated to computer graphics and reverse engineering, which provide tools to compute 3D triangulation out of dense point clouds. The points are considered in the local 3D neighborhood, and therefore triangles can be built one over another, shaping overhung forms. Once the mesh is constructed, it can be exported to various file formats. I use the STL ASCII as the exchange file format, as it contains easily accessible information on each triangle: the normal vector and coordinates of vertices. In order to implement the SSM into GIS, the Python scripting language is used (Fig. 5). In the first step, the STL file is converted into two text files: one of them contains information on geometry of triangles (GENERATE file), while the other one stores the normal vectors coordinates (NORMAL file). The GENERATE file is ready to read into ArcGIS environment by one of the standard procedures, which converts properly formatted text file into the polygonal shapefile (ESRI 2012). As the ArcGIS presents the object-oriented approach, each triangle constitutes one object. Therefore, the fact that some triangles occur one above another, representing overhung parts, is not a problem for GIS software functionality.

The normal vectors coordinates are joined with the shapefile attribute table. The slope gradient and orientation (aspect) are computed for each triangle according to the eq. 5 and 6 and added to the shapefile attribute table.

All the described Python scripts are added to ArcGIS software and placed in the 3D Mesh Toolbox.

The 3D mesh used for the analysis (the extracted and decimated part of the original mesh) was saved into STL file. Using the SSM Read STL tool from 3D Mesh Toolbox, the Kościelec model was imported into ArcGIS software as a polygonal feature class. The normal vector components were added to the feature class attribute table. By means of the SSM Slope and SSM Aspect tools the two parameters were calculated and added to the attribute table.

In order to validate the analysis results and to compare them between TIN and SSM models, a set of randomly distributed points were used. Both models were investigated in the particular locations for slope and aspect values, and statistical analysis of differences between them was conducted.

Results

The outcome of the research was tools implemented into GIS software and results of the test model analysis. The 3D Mesh Toolbox contains three tools. When used together with standard ArcGIS tools, they can be used for Steep Slope Model conversion, import and analysis. The toolbox is ready to be loaded and utilized by any ArcGIS user (available on demand via e-mail).

When comparing the traditional TIN with the SSM, significant differences can be easily observed (Fig. 6). They result from the resolution of both models and from the approach to surface representation – in 2.5D and 3D. First, the various size of the basic reference unit, e.g. triangle, determines size of the features that can be described. This can be considered as simplification and generalization. Second, any of the TIN triangles' slope value exceeds 90°, as in the 2.5D approach the invisible overhung parts do not exist and they are replaced by very steep surfaces. The 3D approach enables true 3D surface representation, so the SSM abounds with overhangs.

The Kościelec mesh, when observed in the orthogonal projection on the XY plane, showed significantly shortened steep slopes. The overhung parts were not visible at all: they should be visualized in an oblique view. Within the western walls of the Kościelec Mt. many triangles were attributed with slope

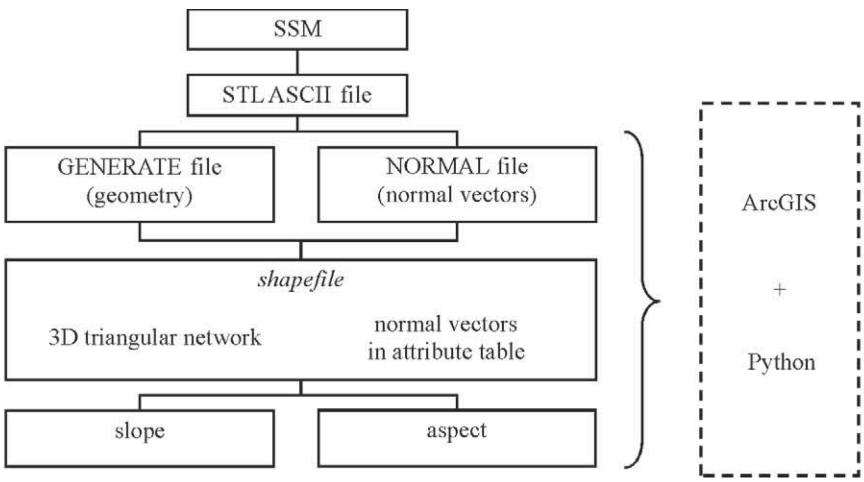

Fig. 5. Steep Slope Model implementation into GIS

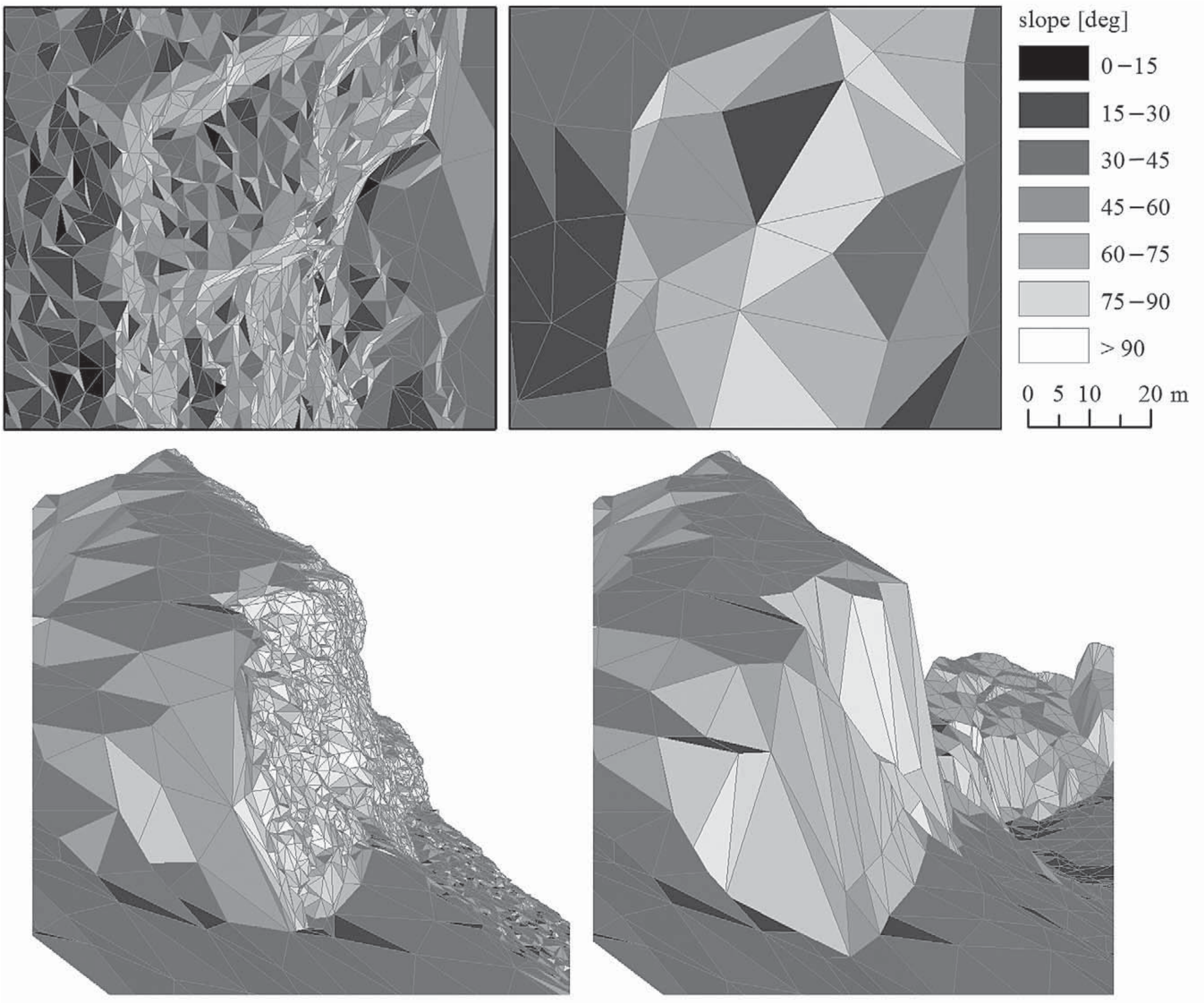

Fig. 6. 3D Mesh Toolbox (ArcGIS)

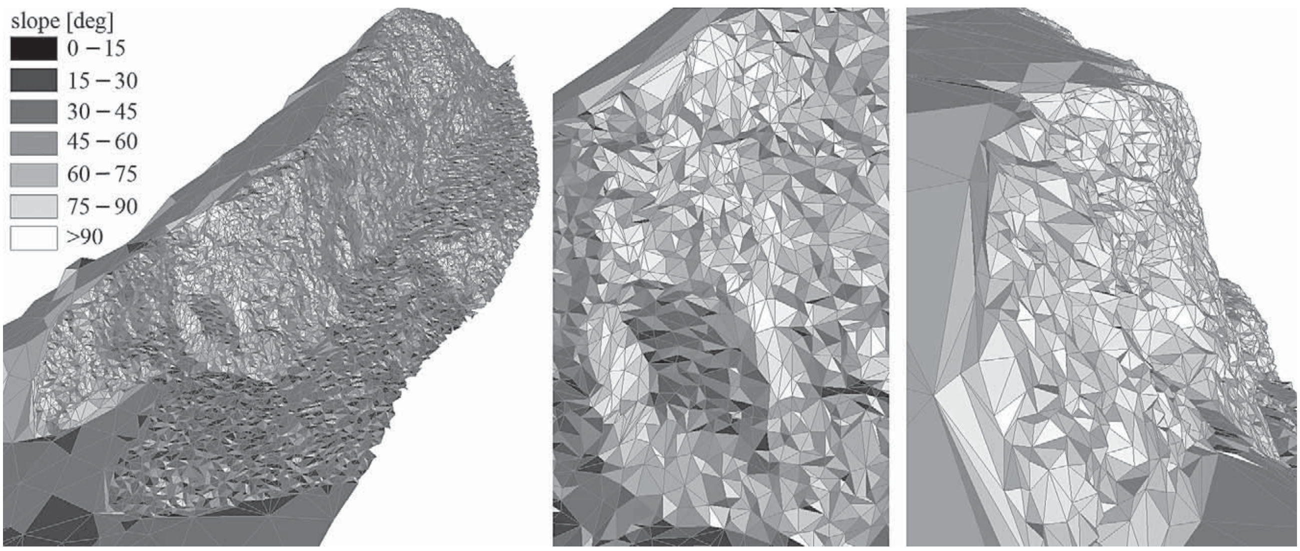

Fig. 7. Slope gradient computed out of SSM, in oblique view

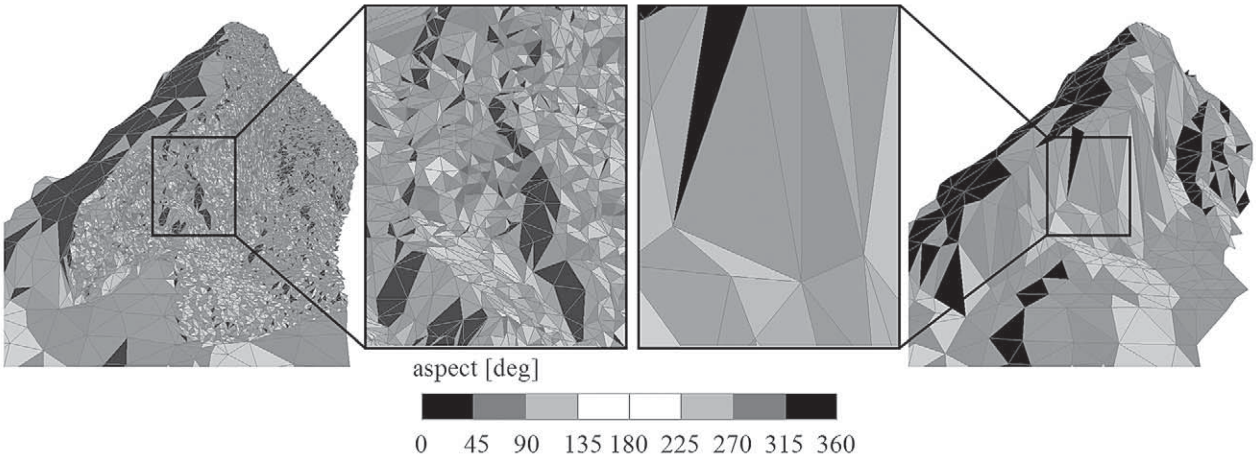

Fig. 8. Aspect computed out of SSM (left) and traditional TIN (right) in perspective view

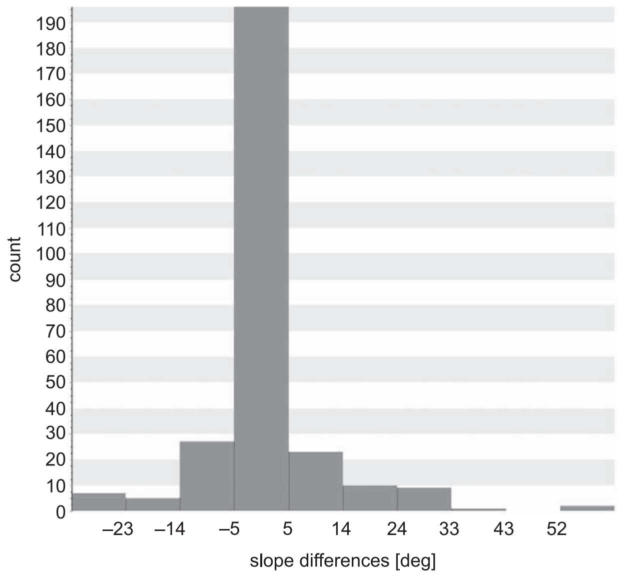

Fig. 9. Differences of slope values computed out of TIN and SSM models

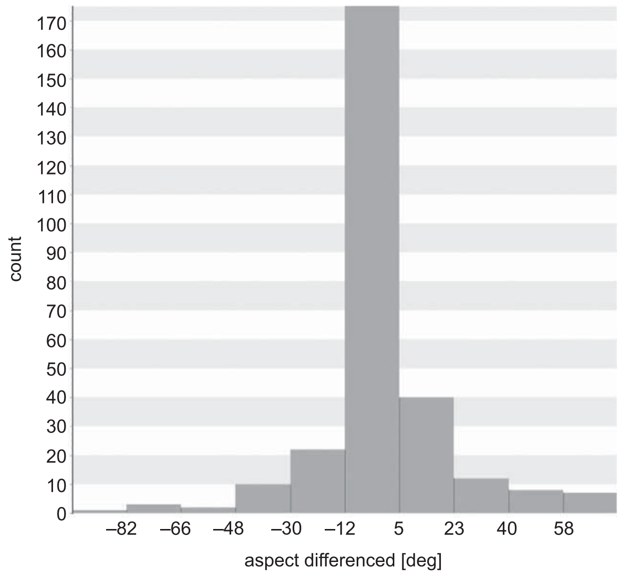

Fig. 10. Differences of aspect values computed out of TIN and SSM models

gradient over 90°, but there was also a large number of small gently sloped triangles. The number of the overhung triangles equalled to 2861, which constituted 14% of the total number of triangles in the mesh. Both slope and aspect values were extremely diverse reflecting sudden changes of elevation at short distances and relief complexity (Fig. 6–8).

This limitation of 2.5D model strongly influences its analysis. The morphometric parameters differ significantly between 2.5D and 3D models. Slope values computed on the base of the TIN model range from 0.0 to 86.6°, whereas the SSM analysis resulted in values from 2.4 up to 165.4°.

A set of 114 randomly distributed points was used to compare the results of slope and aspect analysis, derived from TIN and SSM models. The average difference between slope values computed out of TIN and SSM models equaled to 0.8690° ± 6.7949°, minimum and maximum were –26.4703° and 25.0429°, respectively (Fig. 9). Analogous procedure applied to the aspect values gave following results: the average difference between aspect values computed out of TIN and SSM models equaled to –1.4047° ± 15.4992°, minimum and maximum were –74.5015° and 50.5922°, respectively. So aspect values in particular locations were significantly different (Fig. 10).

Discussion and conclusions

The attempts made by geomorphologists to describe slope shapes and processes lead to better understanding of their evolution. According to the conceptual modelling given by Klimaszewski (1978) and Bishop & Schroder (2004), slope could be characterized by different parts, e.g. segments distinguished along the slope profile. Single profile is continuous information, but only in two dimensions. A set of profiles, created in equal intervals or in characteristic points, is spatially discrete description of the surface. The 3D mesh, used in this study, can represent steep slopes relief continuously, in details, including overhung parts. For that reason several limitations caused by the discrete description can be to overcome.

The tendency to solve many geomorphologic problems by means of new, advanced technologies, leads to GIS and GIS-based concepts. Hodgson & Gaile (1999) emphasized the importance of the conceptual modelling in GIS evolution. Indeed, many new algorithms and methods have been developed since then, but they concern mainly 2.5D DEMs. As the 3D representations and analysis are much less developed than 2.5D (Schön et al. 2009), the standard GIS framework does not support such type of data as the 3D mesh and does not provide tools for their analysis. A foundation for linking the GIS environment with 3D surface model analysis was based on the concept of linear algebra utilization (Corripio 2003). The paper presented that the surface orientation angles representation by means of the normal vector is efficient and can be easily implemented into GIS software.

In an object-oriented approach, surface is built by the triangular network; each triangle constitutes a plane and has a normal vector. The conceptual model is therefore not limited to the 2.5D case where one planar position refers to only one vertical (Z) value, as even the partially overhung 3D mesh can be described in this way.

The developed tools extended abilities of the digital surface models analysis and geomorphometry. Terrestrial data acquisition technologies might be used for the purpose of surface shape reconstruction as they allow better insights into steep slope than aerial sensors. The slope processes and geomorphic hazards, like debris flows, avalanches, rockfalls or landslides (Krzemień 1991, Kotarba 1992, Kotarba & Pech 2002, Rączkowska 2006) can be thus monitored with higher accuracy and documented easily within GI systems using the developed methods. The terrain parameters, which often differ significantly between the TIN and SSM models, are probably more accurate and can be important improvement in geomorphologic research.

Even though, there is still a place for further development of new mathematical solutions and algorithms for 3D surface model analysis that can be used in geomorphology, hydrology, glaciology, etc. (Corripio 2003). First, the slope gradient and aspect are fundamental morphological parameters, but they are not sufficient in most cases. Other parameters including, but not limited to, surface planar and profile curvature, hillshade or viewshed, are necessary. Second, the tools should operate faster and be more efficient. The problem can be solved when the model is stored and indexed in a geodatabase (Schön et al. 2009).

The results of the research lead to the conclusions that the 3D surface analysis based on vector algebra can be very efficient and useful when it comes to monitoring, interpretation and environmental modelling of steep mountain slopes. Moreover, they minimize basic reference unit and preserve the extreme values that can be introduced and analysed in GIS environment.

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