14 Limitations on Predictive Modeling in Geomorphology

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ABSTRACT

Sources of uncertainty or error that arise in attempting to scale up the results of laboratory-scale sediment transport studies for predictive modeling of geomorphic systems include: (i) model imperfection, (ii) omission of important processes, (iii) lack of knowledge of initial conditions, (iv) sensitivity to initial conditions, (v) unresolved heterogeneity, (vi) occurrence of external forcing, and (vii) inapplicability of the factor of safety concept. Sources of uncertainty that are unimportant or that can be controlled at small scales and over short times become important in large-scale applications and over long time scales. Control and repeatability, hallmarks of laboratory-scale experiments, are usually lacking at the large scales characteristic of geomorphology. Heterogeneity is an important concomitant of size, and tends to make large systems unique. Uniqueness implies that prediction cannot be based upon first-principles quantitative modeling alone, but must be a function of system history as well. Periodic data collection, feedback, and model updating are essential where site-specific prediction is required. In large geomorphic systems, the construction of successful predictive models is likely to be based upon discovery of emergent variables and a corresponding dynamics, rather than upon scaling up the results of well-controlled laboratory-scale studies.

INTRODUCTION

Recent efforts at simulating the evolution of large geomorphic systems such as alluvial fans (Koltermann and Gorelick 1992), deltas (Tetzlaff and Harbaugh 1989), hillslopes (Ahnert 1987; Kirkby 1986), fluvial drainage systems (Montgomery and Dietrich 1992; Howard et al. 1994; Willgoose et al. 1991, 1994), and badlands (Howard 1994) rely on the

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implementation of large-scale quantitative sediment transport models. The need for environmental forecasting and for prediction of the future behavior of engineered sediment systems (e.g. Toy et al. 1987; Hanson and Kraus 1989; Riley 1994) is another potential application of quantitative sediment transport models to landscape evolution. Engineering prediction differs from geomorphic or geologic reconstruction in that risks and costs are associated with prediction, that prediction targets the unknown future while reconstruction targets the past, and that time- and space-specific prediction of future landscape configuration may be required, as opposed to generic reconstruction of landforms and landscapes. Reliable prediction would allow us to foresee how landscape features such as watercourses, soils, topography, and vegetation may evolve by their own dynamics, or in response to climate change or tectonic activity, or as the result of modem disturbances such as grazing, bulldozing, clearing of vegetation, military operations, construction of highways and utility corridors, and strip mining. Engineering prediction often involves shorter time scales than geologic reconstruction, but predictive time spans of thousands of years or more are of interest, especially where large-scale surficial disturbances occur (e.g. on military bases), or where hazardous materials are involved (as at the Ward Valley, California, nuclear waste site). Increasing use of landscape evolution codes in the geomorphic literature may presage widespread use of these methods for engineering and environmental purposes. The risk-associated context in which these methods will be used suggests that the time is appropriate to examine limitations that are likely to apply to predictive geomorphic modeling.

Several authors have recently addressed critically the nature of prediction in the geosciences and in geological engineering (Tetzlaff 1989; Oreskes et al. 1994), with special attention to fluvial geomorphology (Baker 1988, 1994a, b), coastal engineering (Smith 1994; Pilkey et al. 1994), and hydrology (Konikow 1986; Anderson and Woessner 1992; Konikow and Bredehoeft 1992; Rojstaczer 1994). Schumm (1985, 1991) has discussed distinctive features of geomorphic systems such as singularity (uniqueness) and sensitivity that make prediction of future behavior difficult in geomorphology. The present chapter focuses on the connection between quantitative constitutive models or relations and prediction of specific sediment transport behavior in large-scale (e.g. fluvial, coastal, and hillslope) geomorphic systems. Constitutive relations are defined at the smallest resolved scale of the problem and summarize our knowledge (or assumptions) about system behavior below the chosen level of resolution. One can attempt to derive constitutive rules from controlled experiments (laboratory-scale) in terms of basic physical quantities like grain size and shear stress that appear to be important determinants of transport. Alternatively, one can invoke rules that are not directly based on the underlying physics and then attempt to justify their selection by showing that certain aspects of the dynamics of geomorphic systems appear to be consistent with the choice of such 'emergent' rules. Both of these approaches are discussed below. The issue of predictability cannot, ultimately, be resolved by logic, mathematics, or computer simulation. It can only be addressed by making predictions and then comparing those predictions against the future behavior of real-world geomorphic systems. The small amount of effort given in current geomorphic research to prediction of the future is understandable in view of the long time scales and large spatial scales that are often of interest. Nonetheless, the relative absence of predictive studies, with subsequent confirmation or refutation of results, represents a significant gap in the geomorphic agenda.

Bedload transport is an example of a specific process that has been studied extensively in the laboratory and that also plays an important role in the evolution of many geomorphic systems. Over a period of many decades, small-scale experiments in flumes have been carried out and semiempirical and empirical models developed and refined on the basis of those experiments (ASCE 1975). These models reflect properties such as grain size and surface slope that are underlying determinants of the physical behavior of the system. For example, the Meyer-Peter formula can be written (Meyer-Peter and Muller 1948) (in SI units) as

$$\frac{q^{2/3}S}{d_{50}} = 17 + 0.4\frac{q_s^{2/3}}{d_{50}},$$

where q_s and q are, respectively, sediment and water discharge in kg m^{-1} s^{-1}, S is the local slope, and d_{50} is the median grain size of the bed material in meters. Other models or 'formulas' are expressed directly in terms of bed shear stress. Although empirical, these expressions are derived from experiments where the independent variables are well-controlled. The form of these equations is therefore directly connected to measured physical behavior under specific experimental conditions. Figure 14.1 shows the tightly clustered experimental results upon which the Meyer-Peter formula is based.

When the predictions of formulae due to different authors are compared, however, agreement between them is typically poor. Figure 14.2 shows predictions of bedload transport rates based upon a number of well-known transport equations. These curves were derived under diverse experimental conditions, and experimental variability may account for some of the differences in prediction shown in the figure. However, the transport equations illustrated are intended to be applicable for a range of grain sizes from medium sand to granules. It thus seems fair to make a general comparison, as shown in Figure 14.2, of the transport rates predicted by these equations. This comparison is based upon results of a comprehensive review of sedimentation sponsored by the American Society of Civil engineers (ASCE 1975). Variation over several orders of magnitude between predictions for different models suggests that application of such formulas to large-scale geomorphic systems, where local conditions are often poorly known, will result in significant uncertainty.

In fluvial problems, empirical rating curves that bypass any reference to important underlying physical variables (grain size, shear stress, etc.) have often been used in practice. River transport of suspended and contact load can be described by power-law rating curves that relate transport rate to total discharge (ASCE 1975; Richards 1982). Rating curves are also commonly used to estimate reservoir sedimentation (Singh and Durgunoglu 1992) and soil erosion (Wischmeier 1976). Such rating curves do not reflect underlying small-scale properties of sediment transport, but are keyed to measurable large-scale properties such as total discharge and average slope.

These examples reflect engineering attempts to make predictions of geomorphic processes. Such attempts may seem crude by the standards of scientific geomorphology, but geomorphic prediction is in practice nearly always based on empiricism. This is partly due to immaturity of the scientific basis of geomorphology, but it also reflects the fact that complex systems such as those characteristic of geomorphology tend to be resistant to reductionism. Because of its overwhelming success in physics, one is accustomed to

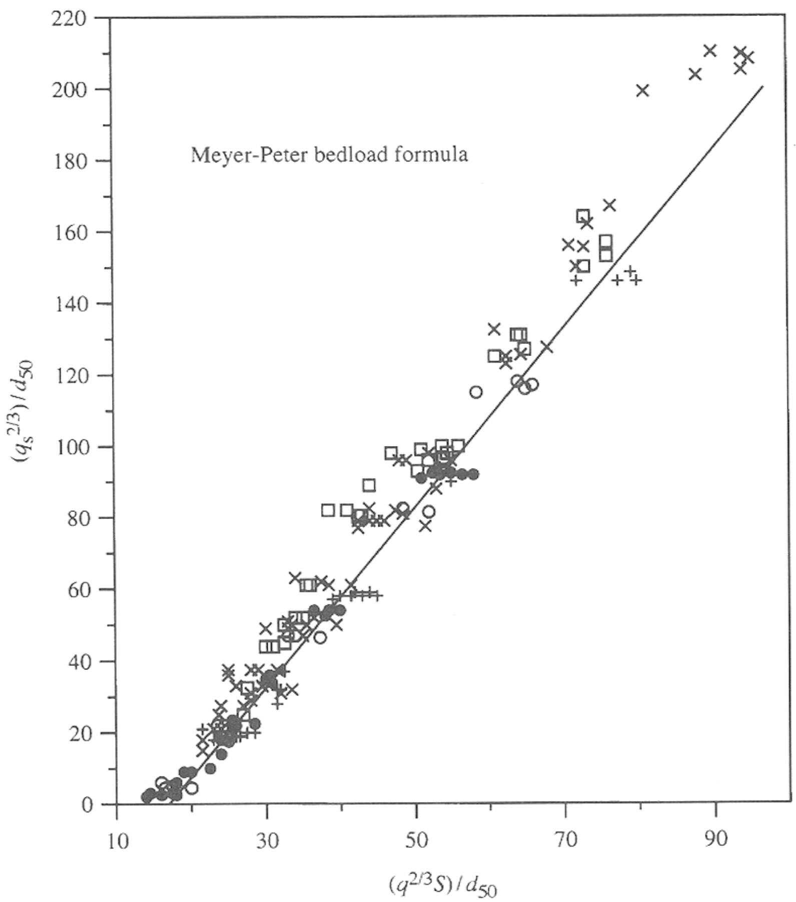

Figure 14.1 Sediment transport rate $q_s^{2/3}/d_{50}$ versus water discharge rate $q^{2/3}/d_{50}$. Data compiled by Meyer-Peter and Muller (1948). S is water surface slope, q_s and q are, respectively, sediment andwater discharge in kg m^{-1} s^{-1}. The line is the original Meyer-Peter formula
The symbols represent experiments with particles of different mean size d_{50}: 0.0286 m (closed circles); 0.00505 m (open circles); 0.00702 m (plus signs); 0.00494 m (crosses); 0.00317 m (squares)

connecting reductionism with scientific investigation in general. In geomorphic systems, 'empirical' variables that are found to be useful for prediction may in fact be related to emergent variables of the system. In such cases, searching for emergent variables, and the constitutive rules that connect them, should be a central focus of activity of geomorphological science. This point of view has been recently articulated by Werner (1995) in terms of geomorphic attractors.

To return to the case of bedload transport, if a laboratory-tested model such as that due to Meyer-Peter (Figure 14. 1) were an accurate constitutive model, it might in principle be used as the basis for scaling up to large-scale geomorphic applications. Several decades

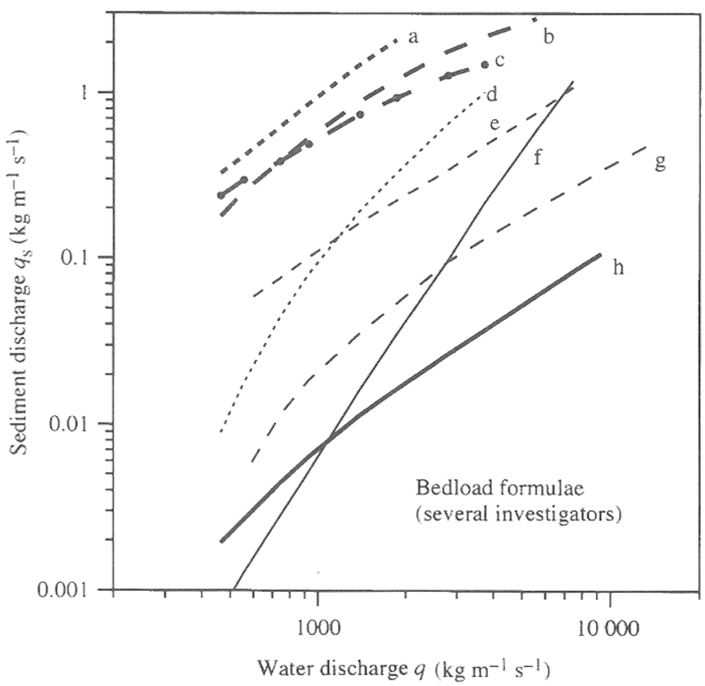

Figure 14.2 Sediment transport rate q_s versus water discharge q for a sandy bed, based on several transport formulas: Shields (a), Einstein-Brown (b) DuBoys (c), Engelund-Hansen (d), Blench (e), Laursen (f), Schoklitsch (g), Meyer-Peter (h). Based on information compiled by the American Society of Civil Engineers (ASCE 1975)

ago, scaling up meant, at most, using local constitutive expressions of sediment discharge rates to predict total stream sediment discharge by multiplying by the stream width (e.g. Graf 1971; ASCE 1975; Yalin 1977). Today, the calculational limitation to upward-scaling is being lifted as numerical simulation approaches based on cell or grid methods (e.g. Tetzlaff and Harbaugh 1989) allow the integration of a large number of pieces of local information. The simulation studies mentioned above of fans, drainage networks, and other geomorphic features are direct consequences of this relaxation of earlier constraints on modeling. An example in engineering is the impending replacement by the US Department of Agriculture of the Universal Soil Loss Equation (Wischmeier 1976) (a rating curve approach) as the basis for estimating potential erosion on agricultural fields and rangelands. A detailed mechanistic, computer-based soil loss model, the Watershed Erosion Prediction Project (WEPP) (Lane et al. 1993) will be used instead. The use of computer-based methods in attempts to predict the future behavior of sediment transport systems can be expected to continue to increase. Potential limitations on the predictive power of such approaches, especially where long time scales are involved, is the focus of this chapter.

In moving from studies of sediment transport at small spatial scales and short time scales to large-scale geomorphic applications, several sources of uncertainty or error arise that affect accuracy of prediction. These uncertainties provide limits on how effectively one can expect to use knowledge of the basic physical processes of sediment transport to make useful large-scale predictions in geomorphology. Sources of uncertainty and error are discussed below.

MODEL IMPERFECTION

Incremental 'improvement' in sediment transport models at the laboratory scale will not necessarily add to our ability to make predictions of sediment transport at a large scale. For example, the explicit introduction of grain-size fractions as variables to be used in place of mean grain size would seem to allow in principle greater predictability of bed evolution in poorly sorted sediment. But the uniqueness of each natural sediment bed in terms of vertical and lateral variations in grading, and the difficulty of measuring such variability, make model implementation increasingly difficult as the model becomes more 'realistic'. For such reasons, quantitative geomorphic studies of large-scale evolution are not usually based on models derived directly from laboratory-scale studies of sediment transport rates, such as those shown in Figures 14.1 and 14.2. Instead, expressions for sediment flux q_s are parameterized in terms of more easily determined variables. Willgoose et al. (1991) choose $q_s \propto q^m S^n$, where q is local water discharge (averaged over a cell), S is local slope (also averaged over a cell), and m and n are parameters that in principle can vary spatially to represent heterogeneity in surface conditions.

Although in some cases large-scale power-law rules can be shown mathematically to result from suitable averaging of small-scale power-law transport formulas, such a procedure cannot be carried out in practice for each large-scale application because of lack of knowledge of the fine-scale detail necessary to implement the averaging. Moreover, in general, averaging of a small-scale power-law rule will not result in a power-law rule for averaged variables, except in special circumstances, because the transport rules are nonlinear. Essentially one must search for new, higher-level rules that emerge at the large scale. In practice, the applicability of a sediment transport rule to be used at the large scale must be determined on the basis of its effectiveness in specific applications at that scale. One cannot expect that these rules are normally reducible to or explicitly derivable from laboratory-scale transport formulas. It may turn out that large-scale power-law rules, such as that given above, are suitable emergent rules for specific applications, but this fact must be established by the large-scale utility of these rules, not by appeal to the form of the small-scale transport formula.

In any case, for bedload transport, both engineering experience and critical studies show that small-scale transport formulas are unreliable, as suggested by Figure 14.2. A report of the American Society of Civil Engineers (ASCE 1975, p. 229) concludes an extended discussion of bedload transport by noting that 'sediment discharge formulas, at best, can be expected to give only estimates'. Graf (1971, p. 156) comments, following an extensive analysis of many bedload formulas, that 'an application of bedload equations to field determinations remains but an educated guess'. More recently, Gomez and Church (1989, p. 1182), on the basis of a detailed statistical analysis of 12 bedload formulas, conclude that 'on the basis of the tests performed by us ... none of the selected formula, and, we guess no formula, is capable of generally predicting bed load transport in gravel bed rivers'. This state of affairs pertains to using bedload models locally, where bed and discharge conditions ought to be best known. Use of such models as the basis for prediction in large-scale applications is even more problematic. Moreover, the models themselves represent only one of a number of sources of uncertainty in modeling sediment transport in large-scale systems, and not necessarily the most important source. Consequently, attempts to reduce model imperfection by introduction of an 'improved' small-scale sediment transport 'formula' are likely to be of only limited effectiveness in improving one's ability to predict at the large scale. Prediction and understanding at a small scale, on the other hand, may well benefit from refinement and development of small-scale models.

OMISSION OF SIGNIFICANT PROCESSES

Incremental improvement of small-scale models of erosion and deposition cannot correct the defect of omitting significant physical processes. The larger the spatial scale of the environmental system and the longer the time scale of interest, the greater is the chance that more than one important process will be present. In their monograph on large-scale simulation of elastic sediment transport, Tetzlaff and Harbaugh (1989) employ a fluvial picture of sediment erosion and deposition. They solve depth-averaged equations for water velocity over a given topographic surface, invoke a sediment transport model (similar to the Meyer-Peter and Muller bedload formula (1948) used in river sedimentation studies), pass sediment from one cell to another, and follow the evolution of topography. One possible application of such models is to alluvial fan building (Tetzlaff and Harbaugh 1989). However, alluvial fans may develop in the presence of nonfluvial transport mechanisms. In a given location, fluvial processes may dominate fan construction, but elsewhere debris flows may be important contributors to the fan building process (e.g. Whipple and Dunne 1992). Debris flows can contain up to 80-90% solids concentration

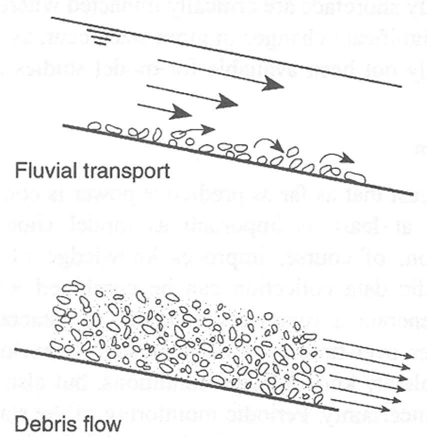

Figure14.3 Bedload normally is composed of a thin carpet of sediment driven forward by the flow of the overlying water, top panel. Debris flows, whose rheology is similar to that of a Bingham plastic, are comprised of an intimate mixture of poorly sorted elastic particles and water, with water content as low as 10%, bottom panel

by weight (Johnson and Rodine 1984). These flows move under rheological conditions (Figure 14.3) that are distinct (Johnson and Rodine 1984; Iverson and Denlinger 1987) from the fluvial conditions that characterize flume studies such as those from which most bedload formulas are derived. If a fluvial erosion and deposition model were applied at a location where debris flow processes are important, then no amount of attention to improvement of the fluvial model can account for the effects of a physically distinct process.

UNKNOWN INITIAL CONDITIONS

Initial conditions are statements about a system that must be made before a model can be implemented. In fluvial transport, these conditions might include the distribution of grain sizes, the cohesiveness of bank material, topographic details of the bed, distribution of channel vegetation, as well as information about stream flow characteristics. Initial conditions also include the distribution of sediment characteristics with depth below the bed surface, since erosion can expose previously buried material. These conditions are known only approximately, or, in some regions of the system, not at all. In studies of coastal sediment transport, essentially all models predicting the shoreface profile along barrier-island dominated coastlines assume that the shoreface is composed entirely of loose sand transportable in bedload or suspension. However, field studies along the North Carolina coast (e.g. Riggs et al. 1995, and references therein) show the existence of numerous premodern features of variable cohesiveness and transportability that underlie or breach the thin veneer of sand (typically < 1 m thick) on the shoreface. Such sediments range from marsh peats and tidal flat muds to indurated sandstones and gravels. The map shown in Figure 14.4, based on side-scan sonar imaging, shows the distribution of silty sand, sandy gravel, and bedrock outcrops along a section of the coast of North Carolina (Thieler et al. 1995). Predictive capabilities of sediment transport models based on the assumption of a uniform, loose, sandy shoreface are critically impacted wherever exposure of cohesive units occurs, or where significant changes in grain size occur, as in the figure. Information of this type has generally not been available for model studies of the shoreface.

Role of Data Collection

These observations suggest that as far as predictive power is concerned, local site-specific data collection can be at least as important as model choice or model refinement. Improved data collection, of course, improves knowledge of initial conditions. More important is that periodic data collection can be combined with model feedback as a practical strategy to generate a running prediction. In general, prediction of specific system behavior diverges over time from actual system behavior. Divergence occurs not only because of incompletely known initial conditions, but also because of the influence of all other sources of uncertainty. Periodic monitoring of the state of the physical system provides the information needed to correct the model (reset the initial conditions) as required. Reliable predictions of specific behavior for essentially all large-scale systems (not just sediment transport systems) require some combination of data collection and feedback. As the values of system variables begin to diverge from predicted values, data

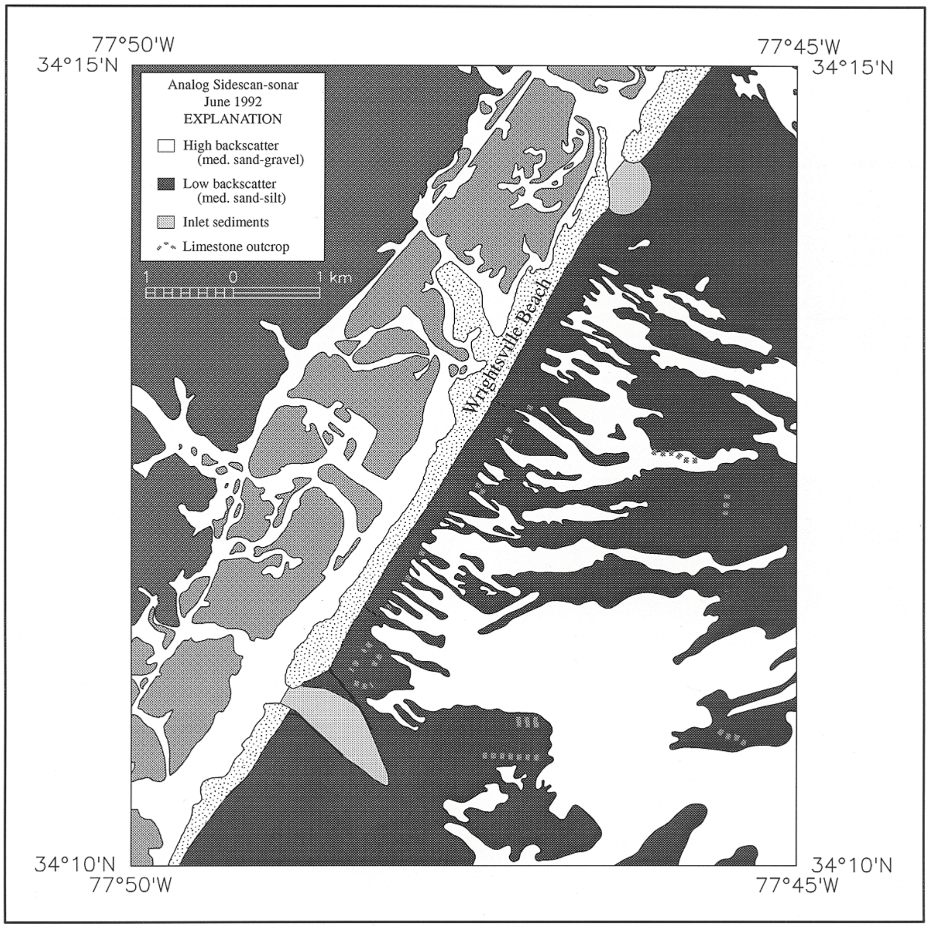

Figure 14.4 Map of sediment cover and bedrock outcrops on the shoreface of Wrightsville Beach, North Carolina. Based on sidescan sonar studies of Thieler et al. (1995). Dark color represents fine sand and silt; light color represents medium sand and gravel; rectangular symbols represent outcrops of resistant Pleistocene deposits. Adapted with permission of Elsevier Science

collection provides information that can be used to correct the model, thus producing a running prediction. Sometimes this procedure is highly quantitative. Kalman filtering techniques have been used successfully to update predictions of water levels for the purpose of optimizing power generation on the Niagara River (Crissman et al. 1993). Here ice jams in the river lead to water level fluctuations that require frequent feedback to 'correct' the model. Frequent measurements of water levels at selected observation stations are used within a hydraulic routing model to update values of discharge and water elevation at each section of the river. Often the procedure is more qualitative. Periodic inspection of dams, aircraft, and other engineered structures for cracks and corrosion or other defects are used as checks to monitor predicted performance over the lifetime of the system. Here, the 'model' is not necessarily a formal mathematical model, but the set of assumptions and relations that are used to predict system behavior. Likewise, large sediment transport systems require data collection and model updating if accurate prediction is to be continuously extended into the future. Thus, the decision to replenish a beach by pumping sand onto the shoreface is determined on the basis of periodic 'data' collection (observation of the state of the beach), not on the basis of long-term model prediction, no matter how, 'realistic' or sophisticated the model might be.

SENSITIVITY TO INITIAL CONDITIONS

In nonlinear systems like those that characterize sediment transport there can exist a sensitivity to initial conditions that effectively prohibits detailed prediction of system evolution. A strong dependence on initial conditions is a highlight of chaotic behavior (Lorentz 1993). Several recent studies have focused on chaotic behavior in geomorphic systems (e.g. Slingerland 1989; Phillips 1992).

Whether a given environmental system is technically chaotic is not always easy to establish (Ruelle 1994). However, models of debris transport and deposition (Tetzlaff 1989), drainage network evolution (Howard 1994; Howard et al. 1994) and stream braiding (Murray and Paola 1994) all show that recalculation of detailed configurations of these systems after a long enough period of time is not usually possible if initial conditions are changed even slightly. Since actual initial conditions are usually known in practice only poorly, errors in prediction are always present in such systems.

These limitations can be important if the required prediction is sufficiently specific. The future pattern of occupation of channel distributaries across a river delta depends strongly on small details of present topography and channel form, and specific prediction is difficult. But, avulsion of the Mississippi River into the channel of the Atchafalaya (Richards 1982; McPhee 1990), a specific event, would have enormous consequences for the city of New Orleans and downstream industrial activity (which would no longer be on the river), and a specific predictive capability for sediment transport and erosion in that reach of the river is highly desirable. Formation of a tidal inlet in the low and narrow region north of Buxton on the North Carolina Outer Banks (Pilkey et al. 1982; Figure 14.5), would lead to isolation of the lower part of Hatteras Island and significantly impinge the economics of the region. A specific predictive capability is desired, but inlet formation by a strong overwash event may depend sensitively on island topography and shoreface bathymetry existing at the time of a given storm, and hence be difficult to predict. On the other hand, if it is the statistical occurrence of avulsion, or tidal inlet formation, or general rather than specific aspects of their dynamics, geometry, or stratigraphy that is the main interest, then sensitivity to initial conditions would not necessarily be an impediment to prediction.

Many features of geomorphic systems are relatively insensitive to initial conditions. Certain channel characteristics of rivers survive large variations in initial conditions, though precise channel patterns can be very sensitive to these conditions. Braided rivers tend to develop on steep alluvial slopes or in watercourses characterized by high sediment load. Meandering streams develop on more gentle slopes and where bank stability is provided by vegetation or cohesive sediment. If model predictions can distinguish quantitatively between conditions required for braiding or meandering, then those pre-

Figure 14.5 Map showing Hatteras Island, North Carolina in the vicinity of the town of Buxton. Formation of an inlet connecting the open ocean on the east to the sound on the west side of the island, in the narrow region north of Buxton (Buxton Overwash Zone), would isolate the town and the southern part of Hatteras Island. Here a site-specific prediction of sediment transport, and of inlet formation, is desireable

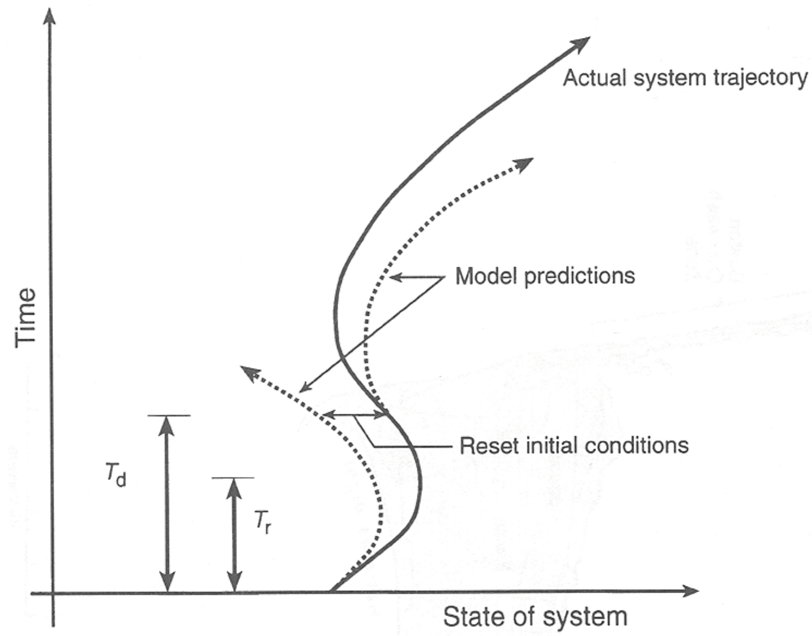

Figure 14.6 The solid curve represents evolution of system through time. Dotted lines show evolution of model prediction, which diverges unacceptably (as determined by the purpose of the prediction) from actual system behavior after a characteristic time T_d. A running prediction will be useful if the divergence time T_d, after which an update of the model is required, exceeds the time T_r, needed to respond to the prediction

dictions, even though inaccurate in the sense that detailed channel forms cannot be reproduced, would clearly be useful. Thus, recent simulation studies of evolution of drainage basins and drainage networks (e.g. Howard 1994; Willgoose et al. 1991) focus appropriately on generic aspects of landscape evolution such as patterns of slope retreat and drainage basin geometry.

The previously noted strategy of data collection and feedback to periodically correct system initial conditions can be applied to help remedy sensitivity to initial conditions. An important factor in such 'prediction-correction' schemes is the relation of the time scale T_d, over which the prediction diverges unacceptably from the actual system trajectory, to the time scale T_r, describing the period of time needed to respond in an appropriate way to the prediction (Figure 14.6). Here T_r, might be the time required to clear a channel by dredging, or to replenish a recreational beach by pumping sand onto the shoreface; T_d is determined by experience with many cycles of prediction and model correction. If $T_r < T_d$, then the model updating scheme can be successful. If T_r, $> T_d$, then the divergence time scale is too short to be useful.

UNRESOLVED HETEROGENEITY

In some physical systems, predictive capability increases with increasing size, because at larger scales one averages over many of the 'details' which make the system so heterogeneous at small scales. In fluid or solid mechanics, the unknown and unknowable detailed motion of molecular constituents can be replaced by appropriate, well-behaved averages. However, not all systems are guaranteed to have useful average properties of this kind. Within the interior of a structural beam in a building, a small averaging volume can be used to define the local state of stress in the beam. But, as the averaging volume is taken larger and larger, eventually exceeding the size of the beam, this approach breaks down. Because a building normally contains heterogeneities of all sizes up to the size of the structure itself, there is no 'typical' averaging volume of the kind one invokes in fluid or solid mechanics, small with respect to the large-scale system, but large compared with the inherent system heterogeneities (e.g. molecules). There is no small-scale physical model (constitutive law) that can be used to predict the detailed behavior of 'buildings'.

Likewise, in large-scale geomorphic systems, it may also be impossible to define a meaningful averaging volume. Heterogeneities appear in the form of variations from place to place in factors such as vegetative cover, soil type and bedrock exposure, and in the presence of discrete entities such as rills and streams. For small enough cells, the basic underlying transport model (such as one of those in Figure 14.2) may directly provide rules for changing the state of the cell (e.g. gain or loss of sediment). However, in larger cells, if the dynamics internal to a cell are sufficiently heterogeneous, then cell evolution rules are not reducible to constitutive rules derived from basic physical (laboratory-scale) principles of sediment transport.

Even small cells can be heterogeneous. Figure 14.7 shows several potential sources of heterogeneity in a dry watercourse. The field of view is about 10 m across in the middle ground. Local variations of flow depth caused by surface irregularities lead to a corresponding distribution of shear stress. Since the bedload sediment transport rate q_s, is a strong function of shear stress τ (e.g. the Einstein-Brown formula (ASCE 1975) gives approximately $q_s \sim \tau^3$), the mean transport rate is not equal to the transport rate calculated on the basis of the mean depth or stress (Baird et al. 1993). Figure 14.8 shows the sediment transport rate q_s calculated on the basis of the Einstein-Brown formula, as a function of slope for flow in a channel of given width and depth. This greatly exceeds the sediment transport rate $q_s(\bar{\tau})$ computed on the basis of the average stress $\bar{\tau}$, where in this case averaging is performed over a swath about twice as wide as the channel.

Other heterogeneities appear in Figure 14.7. Vegetation, whose occurrence is correlated with small changes in elevation, grows preferentially on low terraces lying a few decimeters above the active watercourse. Low flows are little influenced by the presence of plants. At higher flows, the presence of vegetation creates extra flow resistance, roots resist erosion, and sediment and other debris are captured behind vegetative obstructions. Moreover, the land surface above the low-flow channel has a different roughness, a larger mean clast size, and in places a soil crust that is lacking in the low-flow channel. On the left side of the figure, a low bluff several meters high deflects flow, and undergoes parallel retreat by spalling thin slabs of consolidated alluvium into the channel. Significant heterogeneity within an area less than 10 m on a side points to the challenge of eliciting simple emergent constitutive rules for cells in which several processes contribute to intracell dynamics. Bedload transport rules of the kind discussed earlier would be applicable (if at all, given Figure 14.2) on scales much smaller than that of Figure 14.7. Moreover, if intracell heterogeneities are present, the time evolution of the internal state of each cell becomes itself a nonlinear problem that needs to be solved. Internal cell evo-

Figure 14.7 Heterogeneity in surface elevation, clast size, and vegetative cover is present at several scales in this dry watercourse (Greenwater Valley, California). Field of view is about 10 m in the middle ground

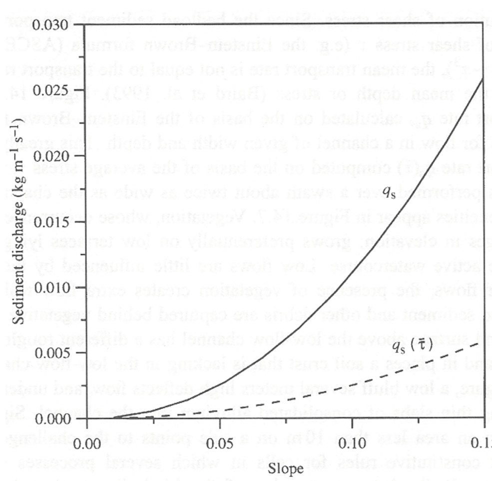

Figure 14.8 Comparison of sediment transport rate q_s in kg m^{-1} s^{-1} computed on the basis of Einstein-Brown formula for flow in a channel of width 0.475 m, depth 0.0225 m, and discharge Q = 6.8 x 10^{-3} m^2 s^{-1} with sediment transport rate $q_s(\bar{\tau})$ computed for the same flow rate but by averaging the bed shear stress over a width of 1 m

lution, however, must be captured in simple rules that can be stated a priori. The rules must be simple because no significant computational resources can be devoted to calculation of intracell dynamics. Such resources have already been used up in creating fine-scale system resolution.

This does not necessarily mean that appropriate constitutive rules cannot be found for heterogeneous cells, but the rules need to be 'discovered' at the cell level, rather than derived from fundamental principles of sediment transport. This is the origin of the power-law-type transport rules used in geological reconstructive modeling and landscape simulation, as discussed above. In such studies, cell rules are invoked that are effective in creating final-state landscapes and stratigraphy that resemble existing landscapes and stratigraphy. These rules are not derived directly by averaging over fundamental physical-process models. For example, the sediment discharge model used as a basic cell rule by Tetzlaff and Harbaugh (1989) in their simulation of large-scale fluvial erosion and deposition resembles the well-known Meyer-Peter and Muller (1948) bedload formulas, expressible as a power law of mean water discharge or excess shear stress. However, water discharge or shear stress variations within a cell, not to mention variations in surface roughness and grain size, will clearly be substantial for most applications, and 'averaging' is never carried out explicitly. The simulations of Koltermann and Gorelick (1992), who studied alluvial fan deposition over a period of 600 000 years using the Tetzlaff and Harbaugh (1989) approach, were performed with a horizontal cell resolution of 120 m. The cell-level sediment transport model used in such studies, therefore, represents essentially a new rule postulated, and hopefully confirmed, at a scale much greater than that at which any physical bedload formula has been, or could be, derived and tested. Such transport rules are not 'averaged' results of laboratory-scale bedload formulas, but should be regarded as new discovered or emergent rules, to be tested, and then used or discarded on their own merits. Thus, rules for large-scale applications should be chosen on the basis of known constraints, such as mass conservation, dependence on variables thought to be important (such as local discharge rates) and on the basis of calibration (fit) to specific field-scale studies. Such rules are not based upon fundamental transport physics, but upon observations (field collection of data) and experience with the modeling requirements of the landscapes of interest.

EXTERNAL FORCING

External forcing arises in an open system where mass, energy, and momentum can enter and be discharged through the system boundaries. Like some of the other sources of uncertainty, external forcing becomes an increasingly important factor in prediction as system size increases. While laboratory experiments are usually carried out at a scale where isolation from external events is possible, large systems are always exposed to the vagaries of nature such as storms and climate change. In fluvial sediment transport, external forcing may be due to increases of discharge resulting from storms or dam releases, to injection into the mainstream of quantities of water and sediment from side channels and slopes falling outside the model boundaries, to backwater effects due to stream impoundment or rising sea level, to tectonic uplift, and to base level lowering. The

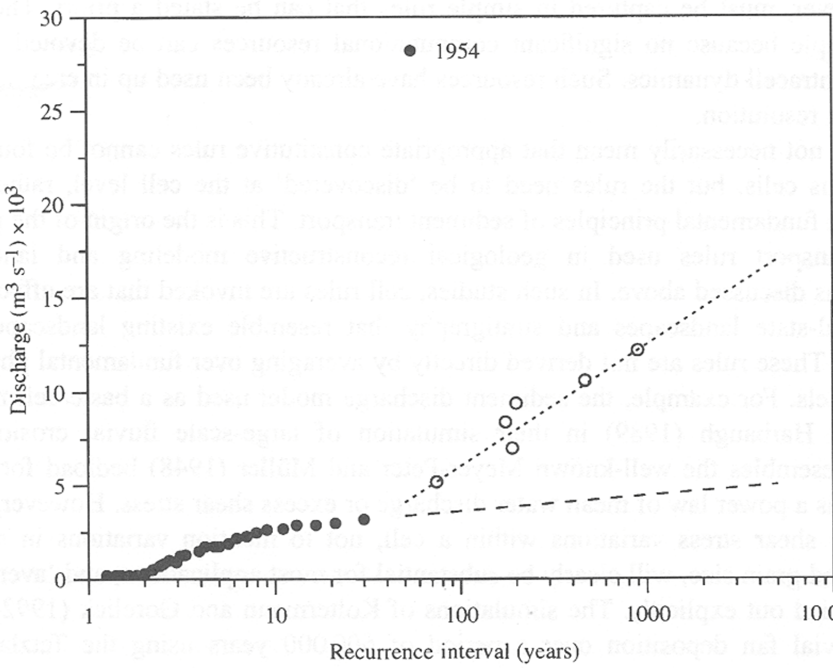

Figure 14.9 Flood frequency data for the Pecos River near Comstock, Texas, (adapted from Patton and Baker 1977) as an example of stochastic external forcing. Extrapolation (dashed line) of historical flood-frequency record (solid circles) suggests a rather different pattern of long recurrence-time events than does geological evidence based on reconstruction of paleofloods (open circles). The isolated 1954 flood data point on the upper part of the figure underscores the risk of basing statistical projections on a limited sample size

predictive capacities of any model are limited if unpredictable external forcing can occur. Sometimes, as in a dam release or impoundment, forcing may be anticipated. In other cases, as in forcing due to storms, it may be anticipated only statistically. A statistical treatment of forcing is possible only if the distribution of events is known. Patton and Baker (1977) discussed historic flood frequencies on the Pecos River in Texas (Figure 14.9). The historical flood record (solid circles) in Figure 14.9 might be extrapolated according to the dashed line to provide an assumed long-term distribution of events upon which prediction of future flood and sediment transport might be based. The data point for the massive flood of 1954, and the reconstructed flood discharges determined by paleohydrological studies (open circles) indicate the pitfalls of this approach. To the extent that a suitable distribution of forcing events is available (or a range of possible outcomes is specified), prediction may be possible, with the frequency of forcing, $1/T_f$, playing a role somewhat similar to the divergence frequency, $1/T_d$, described earlier. However, if forcing is due to occurrence of an event of a kind not anticipated, for example a shift in vegetative cover due to drought climate change, or change in land use, then one loses even a statistical basis for prediction.

INAPPLICABILITY OF THE FACTOR OF SAFETY CONCEPT

In engineering projects that affect property, life, or incur large costs, the risk of a serious mismatch between prediction and outcome can often be protected against by incorporation of a factor of safety in engineering design. The factor of safety helps to protect against a faulty design prediction and against larger than expected excursions of external forcing parameters. Engineered systems are generally constructed of synthesized or carefully selected materials and elements whose properties and interrelationships with each other the engineer (ideally) understands. Thus, elements of a building or bridge are selected and assembled to desired specifications, in a manner over which the engineer has control. If certain elements of the system are not well-understood, or fail to meet specifications, they are rejected and elements or materials substituted whose behavior is acceptable. In short, engineered systems are designed and assembled, and design and human assembly imply a high level of understanding and control. Consequently, model predictions can be effective for such systems, and factors of safety can be included in the design.

For geomorphic systems, however, the material properties and 'construction' of the system (an alluvial surface subject to erosion or deposition, a river channel, a tidal inlet) are not usually well-specified or well-controlled. In their natural state, these systems are not designed and assembled. Where efforts are made to engineer these systems, as in flood control projects, success is due in large measure to the addition of engineered structures (e.g. levees and dams) to the natural system - structures that incorporate a factor of safety. Thus the courses of concrete-lined rivers in the Los Angeles basin are predictable over many flood seasons, and they seldom overflow their banks. Models that are used to predict behavior of a natural system (e.g. how an alluvial surface responds to the forces of erosion following clearing of vegetation) have much less predictive capability than models of typical engineered systems, because the elements of the natural system are not chosen and constructed to predetermined specifications. Environmental systems are highly heterogeneous, material properties are often little understood, and initial conditions are usually poorly known. It is not surprising that a number of famous dam failures, exemplified by the Saint Francis dam (Wiley et al. 1928) and the Vaiont dam (Kiersch 1964) disasters, occurred primarily as the result of initial 'failure' in geologic material adjacent to or upstream of the dam structure, not because the physical (engineered) dam itself failed.

The factor of safety concept is basically a linear scaling method. Where relationships between variables are highly nonlinear, and a clear understanding of the controlling variables does not exist, as in many natural and environmental systems, the factor of safety concept is inapplicable. Lack of a factor of safety results in a reduced probability that a system will behave in a desired (or known) way in the future, and hence leads to a reduced predictive capability.

A substitute for a factor of safety in environmental prediction is the worst-case scenario (Schumm 1985). While the factor of safety depends for its effectiveness on designed and tested construction methods (making a dam thicker), the effectiveness of the worst-case scenario in geomorphic applications depends on the reliability of a model of an undesigned natural system. If the worst-case scenario represents only a modest deviation from the conditions under which the model was derived and calibrated, worst-case predictions are likely to be more useful than when the worst-case scenario represents extreme conditions, for example storm wave attack on a beach. This is where the model will be most susceptible to failure, because of a relative lack of calibration data and because of the increased likelihood of occurrence of unanticipated or neglected processes.

DISCUSSION

The extent to which small-scale sediment transport models that are defined in terms of basic physical quantities such as particle size and bed stress can be scaled up and used as a basis for specific geomorphic predictions is limited by several sources of uncertainty. These include model imperfection, omission of important process, lack of knowledge of initial conditions, sensitivity to initial conditions, unresolved heterogeneity, occurrence of external forcing, and inapplicability of the factor of safety concept. One or more of these sources of uncertainty is likely to arise in any attempt to predict large-scale geomorphic behavior on the basis of scaled-up laboratory-scale studies of sediment transport. Consequently, large-scale geomorphic prediction is difficult to implement in terms of our understanding of the physical behavior of sediment transport at the small scale.

System size seems to be the most fundamental factor that limits predictability in geomorphic modeling. The occurrence of unanticipated processes, the lack of knowledge of initial conditions, the occurrence of external forcing, and the presence of unresolved heterogeneity all become increasingly important with increases in system size. They can often be avoided in systems of small size. Correspondingly, control and repeatability, usually thought of as hallmarks of science in general, are more accurately hallmarks of small size. The fact that large geomorphic systems are often unique, and that control and repeatability are limited or absent, suggests that standard methods of analysis (such as reductionism) that often work well at laboratory scales, may be inapplicable.

Lack of information on initial conditions is a direct consequence of the large size of many geomorphic systems. This lack may be countered to some extent by increases in data collection. Data collection may occur through simple observation, or by input from deployed instrumentation. Updated or corrected predictions can then be obtained as collected data is fed back to the model. From this procedure a characteristic time scale emerges, the divergence time T_d, characterizing the typical period over which the prediction is valid without correction. Frequent data collection will maximize the time over which the prediction is actually usable. Usefulness implies that a characteristic response time T_r must be less than T_d. Data collection, and model updating, are also necessary if one wishes to maximize predictive capabilities in situations where other resources of uncertainty degrade model prediction.

A running prediction is useful if the objective is to produce short-time scale predictions of the evolution of a geomorphic system. Data collection and feedback become less effective as longer-term predictions are sought, since the utility of data collection and feedback in updating the model depends on the experience one gains in applying the model over a number of divergence times. This is how the value of the divergence time is discovered. If the system veers 'prematurely' from the predicted trajectory to an undesirable state, such as excessive erosion at a waste-burial site, an engineered response may be difficult or impossible because of the scale or cost of the problem. Then the prediction has failed its purpose, even though the model can be periodically 'corrected' and a new, more accurate prediction made.

It has been argued that models developed to predict large-scale geomorphic behavior cannot be based simply on upscaled versions of the laboratory-scale sediment transport laws. Reductionism, i.e. explaining behavior at the large scale by appeal to small-scale phenomena, works well with uniform systems, which are usually small themselves. But at large scale, emergent variables that are not (in practice) derivable from basic physics are more likely to be the useful building blocks of predictive modeling. Although not derivable from basic physics, the emergent rules must still obey fundamental physical constraints such as mass conservation. Emergent variables are often used to describe the behavior of large heterogeneous systems in areas other than geomorphology. In biology and ecology, the existence of discrete organisms immediately suggests possible emergent variables. A model of fish schooling might start with the fish itself as the fundamental entity of interest and then attempt to predict schooling patterns from rules of individual fish behavior (Niwa 1994) rather than on the basis of smaller-scale physically based rules such as those describing the internal workings of an individual fish. In geomorphology, identification of emergent variables may be more difficult. In studies of landscape evolution, average water discharge and average slope are two commonly used variables. These are implicitly used as emergent variables, since explicit derivation from fundamental transport laws is not possible, except in the unusual circumstance where local conditions are especially uniform. Put another way, it is a mistake to (necessarily) equate consistency of landforms - for example that shown by the tendency of rivers to follow the rules of hydraulic geometry - with simplicity of the underlying physics. There is great physical consistency in human anatomy and function (e.g. two arms, two eyes, upright posture), but this simplicity has nothing to do directly with the underlying physics. The consistency is instead a manifestation of emergent behavior in extraordinarily complex systems. This is the nature of nonlinear processes that lead to complexity - that simple variables arise at the large scale which are not in practice derivable from the small scale where the simple physics does operate. As far as prediction is concerned, these arguments suggest that in complex geomorphic systems an investment of research effort in improving our knowledge of emergent variables through large-scale modeling and comparison with large-scale phenomena may be more useful than a similar investment of effort in developing small-scale, physically based constitutive models.

A further consequence of size is the uniqueness of geomorphic systems. Large systems are not replicable, as laboratory and most engineered systems are, nor are their constitution, composition, and structure designed or controlled. Two landscapes may be similar in many respects, but will always differ in particulars. Particulars can often be averaged away in small uniform systems (such as molecular positions in a glass of water), but in large systems they persist as heterogeneities (resistant soil layers, patterns of vegetation) that may significantly influence the course of geomorphic evolution. As a consequence, each geomorphic system must be approached anew and analyzed for its specifics and peculiarities, as well as for its similarities to other systems. One should expect no automatic predictive formula to apply to these systems. Instead, experience with similar systems and attention to the historical (geological) record can help inform of conditions and behavior that need to be accounted for in any attempt at modeling. The presence of abandoned braided channels in regions where modem streams and rivers are meandering may indicate the presence of an environmental driving force (such as climate or vegetative cover) that must be accounted for in long-term landscape evolution. These historical factors are largely absent in laboratory-scale experiments, where physical determinants are dominant, but such factors do (or should) provide the starting point from which large-scale geomorphic modeling is initiated and the framework within which it is performed. An important and practical, if obvious, conclusion is that large-scale geomorphic modeling cannot be effectively pursued on the basis of engineering and physical knowledge alone, but must be conditioned by historical and geological understanding. This observation has important implications for the training of engineers and scientists whose future activities will involve use of large-sale geomorphic models.

Finally, many of the conclusions regarding the predictive nature of modeling in geomorphology appear applicable to modeling in the environmental sciences in general. The problems that affect prediction in geomorphology, the ubiquitous occurrence of unknown initial conditions, external forcing and unresolved heterogeneity, and the corresponding inability of reductionism to produce large-scale predictions, are generic problems of size, not of a particular system. The same general approaches that pertain to geomorphic modeling - data collection, model updating through feedback, discovery of emergent variables, avoidance of reductionism, and attention to history - may also be expected to be necessary components of large-scale environmental modeling in general.

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REFERENCES

Ahnert, F. 1987. Approaches to dynamical equilibrium in theoretical simulations of slope development, *Earth Surface Processes and Landforms,* **12**, 3-15.

Anderson, M.P. and Woessner, W.W. 1992. The role of the postaudit in model validation, *Advances in Water Resources,* **15,** 167-173.

ASCE 1975. *Sedimentation Engineering,* edited by V.A. Vanoni, American Society of Civil Engineers, Manuals and Reports on Engineering Practice, No. 54, American Society of *Civil* Engineers, New York, 1975.

Baird, A.J., Thornes, J.B. and Watts, G.P. 1993. Extending overland flow models to problems of slope evolution and the representation of complex slope-surface topographies, in *Overland Flow,* edited by A.J. Parsons and A.D. Abrahams, Chapman and Hall, New York, pp. 199-223.

Baker, V.R. 1988. Geological fluvial geomorphology, *Bulletin of the Geological Society of America,* **100**,1157-1167.

Baker, V.R. 1994a. Geological understanding and the changing environment, *Transactions of the Gulf Coast Association of Geological Societies,* **44**, 13-20.

Baker, V.R. 1994b. Geomorphological understanding of floods, *Geomorphology,* **10**, 139-156.

Crissman, R.D., Chu, C-L., Yu, W, Mizumura, K. and Corbu, I. 1993. Uncertainties in flow modeling and forecasting for Niagara River, *Journal of Hydraulic Engineering,* **119**, 1231-1250.

Graf, W.H. 1971. *Hydraulics of Sediment Transport,* McGraw-Hill, New York, 513 pp.

Gomez, B. and Church, M. 1989. An assessment of bed load sediment transport formulae for gravel bed rivers, *Water Resources Research*, **25**, 1161-1186.

Hanson, H. and Kraus, N.C. 1989. *GENESIS: Generalized Model for Simulating Shoreline Change,* Tech. Report CERC-89-19, Coastal Engineering Research Center, Waterways Experiment Station, US Army Corps of Engineers, Vicksburg, Miss.

Howard, A.D. 1994. A detachment-limited model of drainage basin evolution, *Water Resources Research,* **30**, 2261-2285.

Howard, A.D., Dietrich, W.E. and Seidl, M.A. 1994. Modeling fluvial erosion on regional to continental scales, *Journal of Geophysical Research,* **99**, 13971-13986.

Iverson, R.M. and Denlinger, R.P. 1987. The physics of debris flow - a conceptual assessment, in *Erosion and Sedimentation in the Pacific Rim, Proceedings of the Corvallis Symposium,* edited by R.L. Beschta, International Association of Hydrological Sciences Publ. No. 165, Wallingford, Oxfordshire, UK, pp. 155-165.

Johnson, A.M. and Rodine, J.R. 1984. Debris flow, in *Slope Instability,* edited by D. Brunsden and D.B. Prior, Wiley, Chichester, pp. 257-361.

Kiersch, G.A. 1964. Vaiont reservoir disaster, *Civil Engineering,* **34**, 32-39.

Kirkby, M.J. 1986. A two-dimensional simulation model for slope and stream evolution, in *Hillslope Processes,* edited by A.D. Abrahams, Allen and Unwin, London, pp. 203-222.

Koltermann, C.E. and Gorelick, S.M. 1992. Paleoclimatic signature in terrestrial flood deposits, *Science,* **256**, 1775-1782.

Konikow, L.F. 1986. Predictive accuracy of a ground-water model - lessons from a postaudit, *Ground Water,* **24**, 173-184.

Konikow, L.F. and Bredehoeft, J.D. 1992. Ground-water models cannot be validated, *Advances in Water Resources,* **15**, 75-83.

Lane, L.J., Nearing, M.A., Laflen, J.M., Foster, G.R. and Nichols, M.H. 1993. Description of the US Department of Agriculture water erosion prediction project (WEPP) model, in *Overland Flow,* edited by A.J. Parsons and A.D. Abrahams, Chapman and Hall, New York, pp. 377-391.

Lorentz, E. 1993. *The Essence of Chaos,* University of Washington, Seattle, 227 pp.

McPhee, J. 1990. *The Control of Nature,* Noonday Press, New York, 272 pp.

Meyer-Peter, E. and Muller, R. 1948. Formulas for bed-load transport, in *Proceedings of the 2nd Meeting of the International Association for Hydraulic Structures Research,* Stockholm, pp. 39-64.

Montgomery, D.R. and Dietrich, W.E. 1992. Channel initiation and the problem of landscape scale, *Science,* **225**, 826-830.

Murray, A.B. and Paola, C. 1994. A cellular model of braided rivers, *Nature,* **371**, 54-56.

Niwa, H.-S. 1994. Self-organizing dynamic model of fish schooling, *Journal of Theoretical Biology,* **171**,123-136.

Oreskes, N., Shrader-Frechette, K. and Belitz, K. 1994. Verification, validation and confirmation of numerical models in the earth sciences, *Science,* **263**, 641-646.

Patton, P.C. and Baker, V.R. 1977. Geomorphic response of central Texas stream channels to catastrophic rainfall and runoff, in *Geomorphology in Arid and Semi-Arid Regions,* edited by D.O. Doehring, Publications in Geomorphology, State University of New York, Binghamton, NY, pp.189-217.

Phillips, J.D. 1992. Qualitative chaos in geomorphic systems, with an example from wetland response to sea level rise, *Journal of Geology,* **100**, 365-374.

Pilkey, O.H. Jr, Neal, W.J., Pilkey, O.H. Sr and Riggs, S.R. 1982. *From Currituck to Calabash,* Duke University Press, Durham, NC, 91 pp.

Pilkey, O.H. Jr, Young, R.S., Bush, D.M. and Thieler, E.R. 1994. Predicting the behavior of beaches: alternatives to models, *Littoral 94 Proceedings, Vol. 1,* edited by S. de Carvalho and V Gomes, European Coastal Zone Association of Science and Technology, Lisbon, pp. 53-60.

Richards, K. 1982. *Rivers: Form and Process in Alluvial Channels,* Methuen, London, pp. 90-91.

Riggs, S.R., Cleary, W.J. and Synder, S.W. 1995. Influences of inherited geologic framework on barrier shoreface morphology and dynamics, *Marine Geology,* **126**, 213-234.

Riley, S.J. 1994. Modelling hydrogeomorphic processes to assess the stability of rehabilitated landforms, Ranger Uranium Mine, Northern Territory, Australia - a research strategy, in *Process Models and Theoretical Geomorphology,* edited by M.J. Kirkby, Wiley, New York, pp. 357-388.

Rojstaczer, S.A. 1994. The limitations of ground water models, *Journal of Geological Education,* **42**, 362-368.
Ruelle, D. 1994. Where can one hope to profitably apply the ideas of chaos?, *Physics Today,* **47**, 24-30.
Schummn, S.A. 1985. Explanation and extrapolation in geomorphology: seven reasons for geologic uncertainty, *Transactions of the Japanese Geomorphological Union,* **6**, 1-18.
Schumm, S.A. 1991. To *Interpret the Earth: Ten Ways to Be Wrong,* Cambridge University Press, Cambridge, England, 133 pp.
Singh, K.P. and Durgunoglu, A. 1992. Predicting sediment loads, *Civil Engineering*, **62**, 64-65.
Slingerland, R. 1989. Predictability and chaos in quantitative dynamic stratigraphy, in *Quantitative Dynamic Stratigraphy,* edited by T.A. Cross, Prentice-Hall, Englewood Cliffs, NJ, pp. 45-53.
Smith, A.W.S. 1994. The coastal engineering literature and the field engineer, *Journal of Coastal Research,* **10**, *iii-viii.*
Tetzlaff, D.M. 1989. Limits to the predictive stability of dynamic models that simulate clastic sedimentation, in *Quantitative Dynamic Stratigraphy,* edited by T.A. Cross, Prentice-Hall, Englewood Cliffs, NJ, pp. 55-65.
Tetzlaff, D.M. and Harbaugh, J.W. 1989. *Simulating Clastic Sedimentation,* van Nostrand Reinhold, New York, 202 pp.
Thieler, E.R., Brill, A.L., Cleary, W.J., Hobbs III, C.H. and Gammisch, R.A. 1995. Geology of the Wrightsville Beach, North Carolina shoreface: implications for the concept of shoreface profile of equilibrium, *Marine Geology,* **126**, 271-287.
Toy, T.J. and Hadley, R.F. 1987. *Geomorphology and Reclamation of Disturbed Lands,* Academic Press, London, 480 pp.
Werner, B.T. 1995. Eolian dunes: computer simulations and attractor interpretation, *Geology,* **23**, 1107-1110.
Whipple, K.X. and Dunne, T. 1992. The influence of debris-flow rheology on fan morphology, Owens Valley, California, *Bulletin of the Geological Society of America,* **104**, 887-900.
Wiley, A.J., Louderback, G.D., Ransome, E.L., Bonner, F.E., Cory, H.T. and Fowler, F.H. 1928. *Report of Commission Appointed by Gov. CC Young to Investigate the Causes Leading to the Failure of the St Francis Dam, California,* State Printing Office, Sacramento.
Willgoose, G., Bras, R.L. and Rodriguez-Iturbe, I. 1991. A coupled channel network growth and hillslope evolution model, 1, Theory, *Water Resources Research,* **27**, 1671-1684.
Willgoose, G., Bras, R.L. and Rodriguez-Iturbe, I. 1994. Hydrogeomorphology modelling with a physically based river basin evolution model, in *Process Models and Theoretical Geomorphology,* edited by M.J. Kirkby, Wiley, New York, pp. 271-294.
Wischmeier, W.H. 1976. Use and misuse of the universal soil loss equation, *Journal of Soil and Water Conservation,* **31**, 5-9.
Yalin, M.S. 1977. *Mechanics of Sediment Transport,* Pergamon Press, Oxford, 289 pp.